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C O N T E N S

Pages

PART TWO

A MACROECONOMETRIC MODEL FOR THE SPANISH ECONOMY

ABBREVIATIONS

AR	Autoregressive process
BAR	BVAR model when any interaction between the series is eliminated
BVAR	Bayesian Vector Auroregressive model
CPI	Consumer price index
D	Public deficit
E	Nominal effective exchange rate
FE1	Forecasting Error for one year
FE2	Forecasting Error for two years
FE3	Forecasting Error for three years
GDP	Gross Domestic Product
GDP*	Real Gross Domestic product of the OECD countries
GLS	Generalised Least Squares
I	Rate of interest
L	Employment
M	Liquid Assets held by the public
MA	Moving Average process
MIN	Model with prior information based on empirical regularities
MIX	Theil’s mixed estimator
OLS	Ordinary Least Squares
SURE	Seemingly Unrelated Regression Equations
UVAR	Unrestricted VAR
VAR	Vector Autorregresive model
VARMA	Vector Autorregresive Moving Average model
W	Compensation per employee

FOREWORD

Decision-making by economic agents in environments of uncertainty requires consideration of the possible future course of relevant variables. Such consideration is particularly important in the case of economic policymakers, since their action or failure to act influences the performance of the economy. Within this framework of uncertainty, macroeconometric models are an important tool for examining future prospects, allowing the impact of different economic policy actions on the main variables to be (at least approximately) gauged. The model presented in this paper is a further exponent of the Banco de España's concern to have suitable tools for making macroeconomic forecasts that may support decision-making.

Given that the uncertainty associated with forecasts is by no means negligible, it would seem vital to characterise it. In this respect, econometric models in which all the variables are determined within the model itself allow the uncertainty associated with predictions to be evaluated. In this paper it has been decided to use such a macroeconomic model so that interrelations between the main variables may be captured and, at the same time, so that objective measures of uncertainty about the projections obtained may be obtained.

More specifically, the macroeconometric model for the Spanish economy set out in this paper is a VAR quarterly model which has been used periodically in the Banco de España over the past three years. The first half of the paper offers the most relevant theoretical results of the VAR methodology, referring to the formulation, specification, estimation, identification and uses of this type of model. This methodology is used with increasing frequency and has become part of the applied economist's habitual tool-kit, essentially for two reasons. First, VAR (and particularly BVAR) forecasting models have gained acceptance as valid prediction instruments. And second, much of the work seeking to interpret economic policy has been undertaken in a structural VAR framework.

The second half presents the model, describing its elaboration and setting out some of its uses. Different aspects of the model's construction are addressed here. The choice of variables is thus reasoned, and the prior information considered, the goodness-of-fit criterion selected and the estimation method followed are all detailed. The results obtained under this model are set against alternative models, making for a more appropriate assessment of its characteristics. The emergence of new requirements as regards the monitoring of the Spanish economy (arising from the alteration of the monetary policy framework or from the pre-requisites for EMU entry) has prompted the creation of instruments based on this model, with the aim of meeting these requirements. The examination of some of these instruments concludes the paper.

For an empirical piece of work used over an extended period, credit must be given to many people. The unwavering support of José María Bonilla and José Viñals is particularly to be acknowledged. In addition, Pilar L'Hotellerie, José Manuel Marqués, María de los Llanos Matea and Javier Vallés took the time to read an initial version of this paper, on which they made various comments and suggestions. Likewise, we wish to express our gratitude to all those in the Banco de España Research Department or from other institutions who, at one time or another, conveyed their comments on a version of this model or its results.

PART ONE

METHODOLOGICAL ASPECTS

INTRODUCTION

The Cowles Commission for economic research was headquartered in Chicago from 1939 to 1955. During this period, and especially in the 1940s, its members laid the foundations for what has been called traditional econometrics. Specifically, the contribution of this Commission to econometrics was twofold: it advocated the use of statistical inference in economics, and it developed simultaneous-equation models to an operational state, addressing their identification, estimation and validation.

For three decades, the Commission's econometric principles defined the profession's framework of consensus and monopolised econometric theory and practice. These principles may be summarised in two key points: 1) the imposition of restrictions in the form of nil values for coefficients (for example, the prior division between endogenous and exogenous variables); and 2) econometric specification based on economic theory. Specifically in the macroeconomic sphere, Klein (1947) marks the starting point for the construction of macroeconometric models of potential use in economic policy decision-making processes. The size of these models got progressively bigger, and they were systematically used to quantify the macroeconomic impact of various scenarios defined in terms of alternative paths for the exogenous variables of the model.

In the second half of the 1970s, two authors each wrote a classical article questioning the uses and principles of construction basic to traditional macroeconometric models: Lucas (1976) and Sims (1980). Both critiques of traditional modelling strategy were so profound that, according to these authors, they warranted the abandonment of such a strategy and the initiation of alternative ones correcting what they considered to be unacceptable aspects of the traditional methodology. In effect, these articles were very influential in the United States, giving rise to the start of the research programmes advocated by their authors. Among these research programmes, the present paper focuses on that advanced by Sims (1).

Sims' proposal departed from a direct critique of the construction methods of traditional models. This may be outlined as follows.

(1) Lucas (1976) gave rise to rational-expectations econometrics.

The validity of the restrictions used to obtain a structural interpretation is crucial if it is sought to defend the existence of a connection between reality and the model used to represent it. Sims considered the restrictions used to identify traditional macroeconometric models to be mostly devoid of credibility (2). Economic theory did not justify them. In reality, his argument continued, theories capable of providing unequivocal restrictions were scant in comparison with the number of variables and equations usually included in traditional models. In particular, the alleged exogeneity of many of the variables was more fictitious than real.

By way of illustration, consider the following econometric model:

$$\begin{aligned} Y_{1t} &= F\,(Y_{1t}, Y_{1t-1}, Y_{1t-2}, \ldots, Y_{2t}, Y_{2t-1}, Y_{2t-2}, \ldots; \delta_F) + u_{1t} \\ Y_{2t} &= G\,(Y_{2t}, Y_{2t-1}, Y_{2t-2}, \ldots, Y_{1t}, Y_{1t-1}, Y_{1t-2}, \ldots; \delta_G) + u_{2t} \end{aligned} \qquad \text{[In.1]}$$

where t indicates time, u_1, and u_2 are the disturbances of the model, δ_F and δ_G are vectors of parameters and, for convenience, the model variables have been separated into a vector Y_1, representing the private sector, and another, Y_2, the vector of the economic policymakers' control variables. This model has an identification failing, as it is not possible to establish which of the two equations corresponds to the behaviour of the private sector and which to that of the economic policymakers. A common practice in traditional modelling to resolve this has been to treat the control vector as exogenous; i.e. to reduce [In.1] to the following restricted specification:

$$\begin{aligned} Y_{1t} &= F\,(Y_{1t}, Y_{1t-1}, Y_{1t-2}, \ldots, Y_{2t}, Y_{2t-1}, Y_{2t-2}, \ldots; \delta_F) + u_{1t} \\ Y_{2t} &= G\,(Y_{2t}, Y_{2t-1}, Y_{2t-2}, \ldots; \delta_G) + u_{2t} \end{aligned} \qquad \text{[In.2]}$$

where the vector Y_1 has been eliminated from the equation G, and the assumption is made that the disturbances u1t and u2t are orthogonal (3). Admittedly, the exogeneity of Y_2 ensures the identification of the F and G equation blocks, but what is very likely involved here is an unwarranted assumption since, possibly, those responsible for the control of Y_2 respond to the private-sector events reflected in the path of Y_1.

Sims argues that when the identification of a model resides on such fragile bases, its implications about the underlying interrelations in the economy can be considered only with difficulty, thereby disqualifying it as an instrument of empirical analysis.

The alternative methodology proposed in Sims (1980) was to specify and estimate macroeconometric models not incorporating prior controver-

(2) This idea had been advanced earlier by Liu (1960).

(3) This assumption is not usual in traditional econometrics.

sial restrictions (4). The proposal actually considered was to specify minimally restricted models in which all the variables of a clear economic content were treated endogenously. The resulting models are known as vector autoregressions (VAR) (5). Imposing in [In.1] the restriction that no elements contemporaneous to the variables situated in the right hand side of equation may feature in the left hand side, we have:

$$
\begin{aligned}
Y_{1t} &= F\,(Y_{1t-1}, Y_{1t-2}, \ldots , Y_{2t-1}, Y_{2t-2}, \ldots ;\ \beta_F) + \varepsilon_{1t} \\
Y_{2t} &= G\,(Y_{2t-1}, Y_{2t-2}, \ldots , Y_{1t-1}, Y_{1t-2}, \ldots ;\ \beta_G) + \varepsilon_{2t}
\end{aligned}
\qquad \text{[In.3]}
$$

Along with the assumptions that the functions $F(\cdot)$ and $G(\cdot)$ are linear and that the vector of stochastic disturbances $(\varepsilon_1, \varepsilon_2)$ is white noise, [In.3] would be the VAR representation of the endogenous variables vector (Y1,Y2), as is made explicit in the following chapter.

The implementation of Sims' methodological proposal soon encountered obstacles that ultimately became sources of discussion and research in the 1980s and 1990s. The first was the extensive parameterisation of VAR models. The second was the absence of a specific identification proposal, so that VAR models were reduced-form models devoid of economic interpretation. Currently, both the problem of degrees of freedom and the problem of identification have been resolved relatively satisfactorily, which has made for a readier dissemination of the VAR methods philosophy. This philosophy departs from the acknowledgement that there is extensive uncertainty about the true economic data-generating mechanism. The immediate consequence of this acknowledgement is that an appropriate modelling strategy should enable such uncertainty to be explicitly incorporated into the model specification process, so as to allow its systematic and objective treatment. And it is precisely this idea which warrants insistence on the parsimony of the restrictions, so that the extraction of the relevant empirical regularities may be tackled by means of an as objective as possible reading of the economic data.

This idea permeates the methods and uses described in the rest of this methodological section (6).

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(4) Nonetheless, from the standpoint of VARMA modelling this may be debatable.

(5) These models became known in the literature following Quenouille (1957).

(6) Canova (1995) offers a comprehensive compilation on the specification and use of these models.

I

VAR MODELS

I.1. Formulation

In its most common formulation, the autoregressive representation of a vectorial stochastic process Y of order n is, for all of t, as follows:

$$\begin{gathered} Y_t = B(L)Y_t + DZ_t + \varepsilon_t \\ \varepsilon_t \sim \text{iid}\ (0, \Sigma) \end{gathered} \qquad [I.1]$$

where $B(L) = \Sigma_{s=0}^{m} B_s L^s$ is a matrix polynomial in the lag operator L (such that $L^S Y_t = Y_{t-s}$), with B_s a matrix of order n x n and B_0 the null matrix, i.e. there are no contemporaneous terms, m denotes the number of lags included in each of the endogenous n components of the vector Y, Z is a vector with d deterministic components and D is a coefficient matrix of order n x d. Lastly, ε is a white noise vectorial process of size n, with a zero mean and a covariance matrix Σ. The name «vector autoregression» arises as a natural one for model [I.1] when it is seen that it relates a vector of variables to its own past. In fact, building on [I.1]:

$$Y_t = B_1 Y_{t-1} + B_2 Y_{t-2} + \ldots + B_m Y_{t-m} + DZ_t + \varepsilon_t \qquad [I.2]$$

Alternatively, the autoregressive representation of the stochastic vector Y can be formulated in the following terms:

$$Y_t = X_{t-1}\beta + \varepsilon_t \qquad [I.3]$$

where

$$\underset{(n \times nk)}{X_{t-1}} = \begin{pmatrix} \bar{X}'_{t-1} & 0' & \dots & 0' \\ 0' & \bar{X}'_{t-1} & \dots & 0' \\ \dots & \dots & \dots & \dots \\ 0' & 0' & \dots & \bar{X}'_{t-1} \end{pmatrix}$$

$$\underset{(k \times 1)}{\bar{X}_{t-1}} = \begin{pmatrix} Y_{t-1} \\ Y_{t-2} \\ . \\ . \\ . \\ Y_{t-m} \\ Z_t \end{pmatrix} \quad ; \quad \underset{(nk \times 1)}{\beta} = \begin{pmatrix} \beta_1 \\ \beta_2 \\ . \\ . \\ . \\ \beta_n \end{pmatrix}$$

Each block of zeros in X_{t-1} is a vector of order k, with k being equal to nm+d, the same as the sub-vectors $ß_i$, i=1, ..., n, which contain stacked the i^{th} rows of the coefficient matrices B_s, s=1, ..., m, and D in the formulation [I.1]. The sign «´» denotes transposition.

As instruments representing stochastic processes, VAR models provide a very general theoretical framework. Granger and Newbold (1986) point out that if there is no restriction on the number of lags (m, which may be infinite) and the possibility that the model coefficients may depend on t is accepted, any stochastic process (whether stationary or not) may be approximated by means of an autoregressive representation (1). This generality, in addition to being in keeping with the relatively unrestrictive spirit of the methodology, converts VAR models into attractive starting points for econometric modelling and into a reference framework that reveals the restrictions actually incorporated into alternative models, since any simultaneous-equation or time-series econometric model may be expressed in the reduced form [I.1]-[I.3].

I.2. The unrestricted VAR model

The UVAR (Unrestricted Vector AutoRegression) model is obtained, given a number of lags, with the representation [I.1]. The qualifying adjec-

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(1) Although the expressions [I.1] and [I.3] do not include the possibility of time variation in the coefficients, this will be considered explicitly at a later juncture.

tive «unrestricted» reflects the fact that the UVAR model includes a bare minimum of restrictions that are needed for it to be operational: the selection of a set of n variables, the specification of the (linear) algebraic relationship connecting them and a set of k parameters that allows the sufficient degree of freedom to be had to generate acceptably appropriate statistical estimators.

The UVAR model has been that predominantly used in VAR methodology applications. There are probably two reasons for this: first, its broad generality as a representative instrument of stochastic processes; and further, the fact that it can be readily estimated via methods with appropriate statistical properties, such as the ordinary least squares (OLS) method, which is examined below (2).

I.2.1. Estimation

Let us take as a reference the linear regression framework, whose notation has been used in the formulation [I.3]. As is well known, the OLS estimator has desirable properties in this framework. Specifically, this estimator is consistent and has a normal asymptotic distribution, being the most efficient linear unbiased estimator. The key question is whether these properties hold in the UVAR framework.

As a response to this question, note that one difference between the UVAR and linear regression frameworks lies in the stochastic nature of the regressors in the first case, and in their deterministic nature in the second. The deterministic nature of the explanatory variables in the linear regression model is that which, on one hand, enables the OLS estimators to be unbiased, the Gauss-Markov theorem being applicable, and, on the other, it facilitates the obtention of its asymptotic properties.

When the regressors are stochastic, more possibilities arise, and it is necessary to distinguish between their stationary or non-stationary nature and consider their relationship to the model's error component. In particular, if the variables of the model considered are stationary and the regressors statistically independent of the error component, the properties of the OLS estimator in the deterministic regression framework, conditional upon the sample observations, will hold.

Admittedly, the variables included in the UVAR model may be stationary; however, its regressors are not independent of the error component.

(2) Broadly, attention will focus on estimation procedures not requiring explicit assumptions about the distribution of stochastic disturbances. This is why, in general, procedures such as maximum-likelihood methods are not going to be considered.

Specifically, the regressors in a UVAR model will be correlated with lagged disturbances. In formal terms:

$$E\,[\varepsilon'_{t-s}X_{t-1}] \neq 0, \quad s \geq 2 \qquad [I.4]$$

Although this feature means that certain properties cease to hold, it is not irremediable. In fact, it is true that: *a)* the disturbance vector of the UVAR model is, by definition, a succession of independent random vectors, and *b)* there is no correlation between the current value of the disturbance and the regressors of the model; i.e.

$$E\,[\varepsilon'_{t}X_{t-1}] = 0 \qquad [I.5]$$

Subject to certain regularity conditions, conditions *a)* and *b)* are sufficient for applying the Mann-Wald and Cramèr theorems [see, for example, Harvey (1981)] which, combined, allow to show that, in a model of stationary variables with stochastic regressors, the OLS estimator retains the same asymptotic properties as in the linear regression framework. A stationary UVAR model can, therefore, be suitably estimated via the OLS method.

In the non-stationary framework, the presence of unit roots and of possible cointegration relationships between the components of the vector Y does not result in a lessening of the asymptotic properties of the OLS estimator of the UVAR model. Sims, Stock and Watson (1990) show that, if the potential cointegration restrictions existing are not taken into account and the model is estimated in levels, this estimator is consistent; and Park and Phillips (1989) and Ahn and Reinsel (1990) demonstrate that it has the same asymptotic properties as the maximum likelihood estimator that incorporates the cointegration restrictions.

A further question arising from a UVAR model estimation process is that of whether the application of the OLS method to each of the n equations of the system involves a loss of efficiency in relation to the alternative of estimating the n equations jointly by generalised least squares (GLS).

Regression theory provides the appropriate framework for responding to this question; more specifically, the valid reference framework is in this case that of seemingly unrelated regressions (SURE). Two standard results of the SURE framework are that GLS estimators and OLS estimators are the same in the absence of contemporaneous correlation between the error components (Σ is diagonal in our notation) or when the set of regressors is the same in the n equations. In either of these two cases, joint estimation does not provide gains in terms of efficiency [see Harvey (1981), for instance].

In a UVAR model, Σ is not usually diagonal, yet all the equations have, by contrast, exactly the same regressors. Under this condition it holds that the OLS and GLS estimators coincide. This result justifies the habitual practice of estimating UVAR systems following a single-equation procedure.

I.3. The Bayesian VAR model

As can be seen in expressions [I.1] and [I.3], the generality of the autoregressive representation is based on its extensive parameterisation. But such generosity in the specification may be excessive, since the number of coefficients grows as a quadratic function of the number of variables included and proportionately to the number of lags of each variable, according to the expression n(nm+d).

The UVAR model described in the preceding section seeks to exploit directly the generality of the autoregressive representation, without any type of additional restriction coming to bear on the lag structure once m is selected. As a result, the model tends to be heavily parameterised. Note, for example, that a UVAR model with five endogenous variables, four lags and a constant term per equation, will contain a total of 105 coefficients.

Heavily parameterised models are not, however, those most suitable for the empirical analysis of macroeconomic data, owing to the fact that macroeconomic information tends to be scant and to contain a high proportion of random variability. The conjunction of a heavily parameterised model with scant sample information that is overly random, along with a method that minimises the distance to the data, causes an *overfit;* i.e. the phenomenon whereby the resulting model reflects, fundamentally, random (noise) rather than systematic (signal) empirical variability.

Against this background, analysts of macroeconomic series wishing to resort to the UVAR framework have to specify small models. Indeed, it is not usual to find UVAR applications with more than five or six variables. This is a truly paradoxical obstacle: as earlier indicated, VAR methodology seeks to be an alternative to traditional econometric modelling by avoiding controversial exclusions, but the UVAR framework is not really an operational alternative to traditional macroeconometric models owing to the fact that its generous parameterisation rapidly exhausts the degrees of freedom available, even in small-sized models, the result being models suffering from over-parameterisation.

The Bayesian dimension of VAR methodology was proposed by Litterman (1980) and Doan, Litterman and Sims (1984), with the aim of offering a solution to the problem of overfit for UVAR models other than that involving resorting to economic theory and statistical tests as sources of exclusion restrictions, an habitual solution in simultaneous-equation econometric models. In keeping with the relatively unrestrictive spirit of the methodology, it was sought to avoid the influence of random variability in the estimation without having to address the choice of model should retain the generality of the autoregressive representation.

The Bayesian solution may be viewed as natural on seeing how unsatisfactory the need to take decisions on exclusion or inclusion is in situations where the analyst never knows for sure beforehand whether the value of a specific coefficient is zero, and where knowledge of the value of the model coefficients is not totally absent, as is habitually the case in econometric analysis. The Bayesian perspective enables such exclusions to be avoided and allows the information available to be expressed more realistically through the allocation of probability distributions to the model's different coefficients.

More specifically, the above-mentioned authors proposed complementing the autoregressive representation with the specification of a prior distribution of the coefficients which, by not being diffuse (whereby any value would have the same probability) nor placing the entire weight on a single value, would offer a reasonable range of uncertainty and could be modified by the sample information when both sources of information were to differ substantially. While the prior information is not excessively diffuse, it will only foreseeably be altered by the systematic variability, not the random variability, thus lowering the risk of overfit.

The implementation of this idea involves formally specifying a probability distribution for the coefficient vector ß and combining it with the representation [I.1]-[I.3]. The resulting model of this combination is called Bayesian Vector AutoRegression (BVAR).

I.3.1. Estimation

Adopting the Bayesian approach, ß is a random vector and not a vector of parameters. This point should be underscored. Traditional econometrics or, generally speaking, non-Bayesian econometrics departs from the assumption of the existence of a vector of genuine parameters. However, Bayesian econometrics does not consider the model coefficients as parameters but as random variables which, as such, have a distribution function. In this respect, characterising the stochastic behaviour of Y_t conditional upon X_{t-1} requires explicit assumptions, both about β and ε_t. In the BVAR framework, the following assumptions are usual:

$$\beta|X_{t-1} \sim N(\bar{\beta}_{t-1}, \Omega_{t-1})$$
$$\varepsilon_t|X_{t-1} \sim N(0, \Sigma) \qquad [I.6]$$

β and ε_t. are independent random variables.

The first two assumptions show that, conditional upon the information available at the start of period t, the coefficient vector β and the disturbance vector ε have a multivariate normal distribution, with the mean and variance specified. The assumption of normality is not unavoidable, but it is desirable. That is to say, what is really sought is a flexible model to incorporate prior information into the analysis, and the assumption of normality enables the appropriate properties of the Gaussian framework to be used.

Let us start by pointing out that, from the Bayesian standpoint, the problem of estimating this econometric model is confined to the problem of applying the Bayes theorem to obtain for all t the posterior distribution of $[\beta|X_{t-1}, Y_t]$ on the basis of the prior distribution of $[\beta|X_{t-1}]$ (3) in [I.6] and of the sample information for moment t. We shall focus first on obtaining the posterior distribution, discussing thereafter the selection of the prior information (4).

Theil's mixed estimation technique [Theil (1971)] provides a suitable framework for obtaining the posterior distribution of the coefficient vector by allowing, first, the different sources of information available (prior and sample in this case) to be combined and, further, for it to be interpretable in Bayesian terms (5). To apply the technique, we first need to express our prior information in the form of dummy observations. Note, specifically, that the distribution in the first line of the expression [I.6] can be expressed as follows:

$$\beta = \bar{\beta}_{t-1} + \eta_{t-1} \qquad [I.7]$$

where

$$\eta_{t-1} \sim N(0, \Omega_{t-1})$$

(3) Strictly, this distribution includes sample information to the moment t-1.

(4) From a strictly Bayesian viewpoint, Σ should also be part of the estimation problem; i.e. the problem should be that of obtaining a posterior distribution for $[\beta, \Sigma \mid X_{t-1}, Y_t]$ on the basis of a prior distribution for $[\beta, \Sigma \mid X_{t-1}]$. The usual procedure in the literature on BVAR models has, however, been that of making conditional upon Σ and focusing on the coefficient vector β. We shall confine ourselves to this framework in this paper.

(5) The posterior distribution may be obtained in alternative ways. For instance, in Ballabriga (1991, 1997) the updating scheme provided by the Kalman filter is used.

As stated, expression [I.7] comprises the set of prior information on the coefficient vector β. The second information set is given by [I.3], which defines the connection between the vector of observables Y_t and β. and which, for convenience, is reproduced below:

$$Y_t = X_{t-1}\beta + \varepsilon_t \qquad [I.8]$$

The disturbance vector ε_t is characterised by the second line of expression [I.6] and is, according to the third line of the same expression, independent of the disturbance vector η_{t-1} in [I.7].

The stochastic linear restrictions [I.7] and [I.8] contain the information on β available in t, and they may be combined as follows:

$$\begin{pmatrix} \bar{\beta}_{t-1} \\ Y_t \end{pmatrix} = \begin{pmatrix} I \\ X_{t-1} \end{pmatrix} \beta + \begin{bmatrix} -\eta_{t-1} \\ \varepsilon_t \end{bmatrix} \qquad [I.9]$$

where

$$\begin{bmatrix} \eta_{t-1} \\ \varepsilon_t \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Omega_{t-1} & 0 \\ 0 & \Sigma \end{bmatrix} \right)$$

Theil's mixed estimator β, β_t^{MIX}, is obtained applying the GLS method to the system [I.9]. The estimator is as follows:

$$\begin{gathered} \beta_t^{MIX} = [\Omega_{t-1}^{-1} + X'_{t-1}\Sigma^{-1}X_{t-1}]^{-1}\,[\Omega_{t-1}^{-1}\bar{\beta}_{t-1} + X'_{t-1}\Sigma^{-1}Y_t] \\ \mathrm{Cov}\,(\beta_t^{MIX}) = [\Omega_{t-1}^{-1} + X'_{t-1}\Sigma^{-1}X_{t-1}]^{-1} \end{gathered} \qquad [I.10]$$

The question now is what is the connection between the estimators in [I.10] and the posterior distribution of $[\beta|X_{t-1}, Y_t]$. And the reply is obtained by means of the Bayesian interpretation of Theil's mixed estimation technique: if [I.7], i.e. the prior information specified, is interpreted as a second sample independent of the sample of observables $[Y_t, X_{t-1}]$ in [I.8]. With the prior information included in the form of a dummy sample in [I.9], we proceed as though our information on β were diffuse. Combining the likelihood of the model [I.9] with the diffuse information of β then gives the posterior distribution, which proves to be approximately normal with mean and variance given by [I.10] [see Theil (1971)]. That is:

$$[\beta|X_{t-1}, Y_t] \sim N(\bar{\beta}_t, \Omega_t) \qquad [I.11]$$

where

$$\bar{\beta}_t = \beta_t^{MIX}$$

$$\Omega_t = \mathrm{Cov}\,(\beta_t^{MIX})$$

It may thus be concluded that, conditional upon Σ, [I.10] is a Bayesian estimator updating procedure. If it is used iteratively for all the sample observations, β_T and Ω_T, , may be obtained, thus completing the estimation procedure from the Bayesian perspective.

I.3.2. Time variation of the coefficients

BVAR methodology has so far been described under the assumption that the coefficient vector β has an invariant distribution over time which successive sample observations allow to be estimated with progressively greater accuracy. Habitually, however, analysts may believe there to be non-linear behaviour in their sample. This belief may be explicitly included in the model, accepting as part of the set of prior information the possibility that the distribution of the coefficient vector β may alter over time.

Time variation is a relatively typical feature of BVAR models which makes their specification more flexible and provides a useful mechanism for detecting potential non-linearities in the sample without having explicitly to model the source of the change.

Although other parameterisations are possible, the most usual way of including time variation in the BVAR framework is by specifying the law of motion of β as a first-order autoregressive process. This law of motion usually suffices to detect potential shifts in the linear structure of the model, further enabling the analysis to be maintained within the Gaussian framework. Indeed, the framework described in the previous section may be readily generalised so as to include this type of time variation. Specifically, the coefficient vector of the model now takes the following form:

$$\beta_t = \begin{bmatrix} \beta_{1t} \\ \beta_{2t} \\ \cdot \\ \cdot \\ \cdot \\ \beta_{nt} \end{bmatrix} \qquad [I.12]$$

where the added time index indicates that the stochastic properties of the vector depend on time. In this context the coefficient vector is a stochastic process with a distribution that is variable over time. As a consequence, the characterisation of the stochastic behaviour of Y_t conditional upon X_{t-1} requires broadening the set of assumptions in [I.6] to take this time dependence into account. The set of assumptions used is as follows:

$$\beta_{t-1}|X_{t-1} \sim N\,(\bar{\beta}_{t-1}, \Omega_{t-1})$$

$$\varepsilon_t|X_{t-1} \sim N\,(0,\ \Sigma)$$

$$\beta_t = S\beta_{t-1} + u_t \qquad [I.13]$$

$$u_t|X_{t-1} \sim N\,(0,\ \phi)$$

$$\beta_{t-1},\ u_t \text{ and } \varepsilon_t \text{ independent}$$

where S and ϕ are nk-order squared matrices whose structure will be specified later.

The prior distribution of $[\beta_{t-1}|X_{t-1}]$ (i.e. the equivalent to the first of the assumptions of the BVAR framework represented by the first equation of [I.6]) is now obtained combining the first three lines of [I.13], giving:

$$[\beta_{t-1}|X_{t-1}] \sim N\,(\beta_{t-1},\ \Omega_{t-1}) \qquad [1.14]$$

where

$$\beta_{t-1} = S\bar{\beta}_{t-1}$$

$$\Omega_{t-1} = S\Omega_{t-1}S' + \phi$$

The analysis of the foregoing section is thus valid with the simple replacement, for the whole of t, of β_{t-1} and Ω_{t-1} by β_{t-1} y Ω_{t-1}, respectively, giving rise to the following updating scheme:

$$\beta_{MIXt} = [\Omega_{t-1}^{-1} + X'_{t-1}\Sigma^{-1}X_{t-1}]^{-1}\,[\Omega_{t-1}^{-1}\beta_{t-1} + X'_{t-1}\Sigma^{-1}Y_t]$$

$$Cov\,(\beta_{MIXt}) = [\Omega_{t-1}^{-1} + X'_{t-1}\Sigma^{-1}X_{t-1}]^{-1} \qquad [I.15]$$

Note, finally, that the framework with time variation generates as a particular case the framework without time variation when S is the identity and ϕ the null matrix, in which case the set of assumptions in [I.13] is identical to that of [I.6], and the updating schemes [I.10] and [I.15] coincide.

I.3.3. Prior information

As mentioned in reference to [I.10], for the updating scheme of the mixed estimator expressed in [I.15] to be operational, it is necessary in the first sample period (t=1) to have an initial specification for the matrix Σ

and the prior distribution relating to [I.14], which by extension requires specifying the matrices S, S, ϕ y Ω_0, along with the vector β_0. This initial specification allows the model's set of prior information to be completed.

The selection of the prior information is, undoubtedly, the most distinctive aspect of the specification process of BVAR models. In principle, this information may adopt different forms and be from various sources, hence its attractiveness. However, in the framework of BVAR methodology the main aim of the information is, as mentioned, to reduce the risk of overfit without diminishing the generality of the representation of the model. What is then involved here is purely instrumental information which, as such, does not seek to be definite on average but to offer a realistic range of data-generating mechanisms among which analysts may select that most appropriate for explaining the variability of their sample data.

In keeping with its instrumental nature, the usual prior information in the BVAR framework is of a statistical-empirical origin, lacking in economic content (6). The intention of such economic «neutrality» from the outset is that the resulting specification should be accepted by a broad range of analysts, irrespective of the fact that they may not concur in their view of what the true structure of the economy analysed is.

The essential part of the prior information is made up of three empirical regularities that are characteristic of time series statistical analysis:

1) The assumption that the best forecast of the future value of a series is its current value (the so-called random walk assumption) satisfactorily approximates the behaviour of many economic series.

2) Recent lagged values of a series usually contain more information on its current value than lagged values more distant in time.

3) The lagged values of a series contain more information on its current value than the lagged values of other variables.

As may be seen from the formal description of the model given in the preceding paragraphs of this section, and in particular from the prior distribution expressed in [I.14], the mechanism selected for including the prior information consists of specifying a multivariate normal distribution. The aim is that the prior information should contain regularities 1) to 3), and the assumption of normality allows them to be included, in addition to providing the analytically desirable Gaussian framework. The most direct means of proceeding is to define the prior distribution expressed in [I.14] in t=0 as a set of independent normal nk distributions, one for each coef-

(6) An exception is the work of Ingram and Whiteman (1994).

ficient of the model, parameterised individually in line with regularities 1) to 3). However, this individualised parameterisation strategy would lead to an overfit, which is precisely what it is sought to avoid.

An alternative strategy involves maintaining the independence assumption among the prior nk distributions, but adding a functional dependence among them all and a limited set of parameters that allow their basic dimensions to be controlled so that they reflect regularities 1) to 3). These parameters are known as hyperparameters in BVAR methodology terminology so as to distinguish them from the term «parameter» used in classical econometrics.

Chart I.1 offers the prior density function for an equation representative of system [I.3] and illustrates how regularities 1) to 3) are included:

— Feature 1) is incorporated specifying a mean equal (or close) to one for the distribution of the coefficient of the first own lag, and equal to zero for the other coefficients.

— Regularity 2) is reflected in the reduction of the variance of the distributions as the lag increases; hence, the more distant the lag is, the greater the probability that its coefficient is zero.

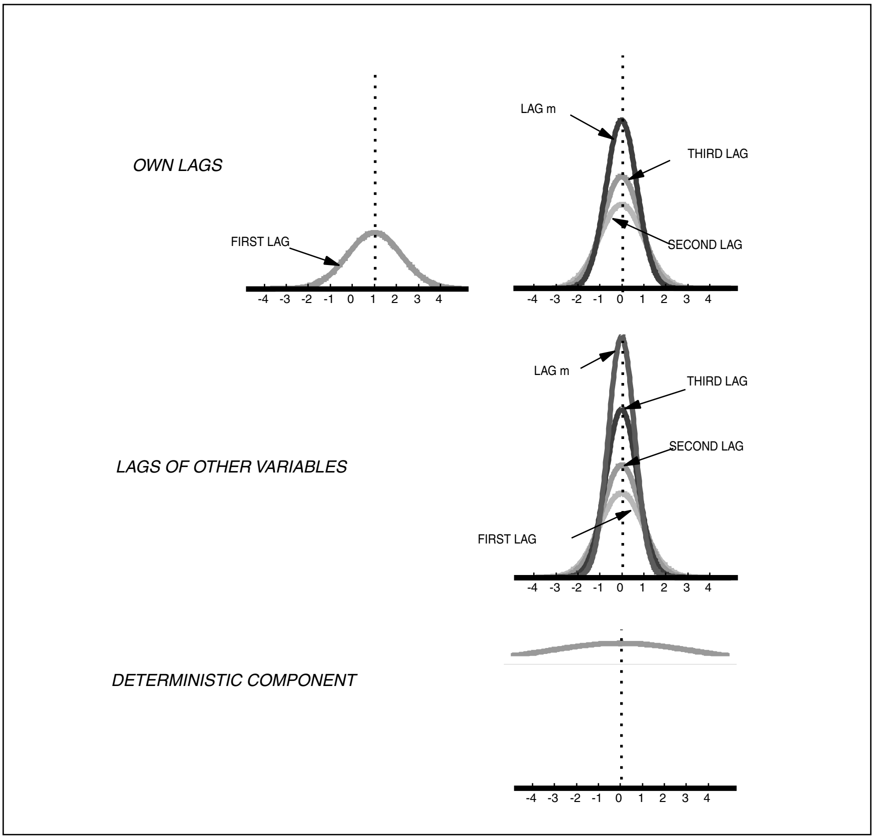
— Finally, characteristic 3) is introduced assigning greater variance to the own lags (row 1 in the chart) than to the lags of other variables (row 2 in the chart), making it more probable that the latter are zero.

As depicted, Chart I.1 also gives some idea of the nature of the set of control hyperparameters. Thus, one of the hyperparameters usually controls the mean of the first own lag coefficient. A second hyperparameter controls the variance of the distributions of the own lag coefficients, and a third that of the coefficients of the lags of other variables. A fourth hyperparameter controls the speed at which the variance of the coefficients (both own and of other variables) diminishes as the lag in question increases. Further, the initial assumption is that the analyst does not have specific information about the deterministic component, whereby the prior distribution for its coefficient is diffuse (row 3 of the chart).

An additional hyperparameter is usually specified to control the overall degree of uncertainty with which the model's coefficients are incorporated. This aspect is crucial for determining the relative weight assigned to the prior and sample information, respectively. In terms of Chart I.1, an increase in this hyperparameter would cause a generalised increase in the variance of the distributions, such that the relative weight of the prior information would be reduced. At the limit, if this hyperparameter took a

CHART I.1

BASIC FEATURES OF THE PRIOR INFORMATION



very high value, the prior distributions of all the coefficients would be diffuse (in terms of Chart I.1 all the distributions would be similar to that presented for the deterministic component in row 3), the prior information would have no weight and, therefore, any Bayesian element would be eliminated.

Admittedly, in specific applications (such as that presented in the second half of this work) the analyst may wish to control other dimensions of the prior information that he deems relevant for the case at hand (e.g. the seasonal or the long-term dimension), but the dimensions described are common to all the applications of BVAR methodology.

Returning to the formal description of the model, these ideas are outlined explicitly below in terms of the elements defining the prior distribution of the coefficients expressed in [I.14].

Starting with the vector β_0,its specification is as follows:

$$\bar{\beta}_{i0} = \begin{bmatrix} 0 \\ . \\ . \\ . \\ \tau_1 \\ . \\ . \\ . \\ 0 \end{bmatrix}; \quad i = 1, 2, \dots, n \qquad [I.16]$$

where the hyperparameter τ_1 occupies the i^{th} position and represents the prior mean of the coefficient of the first own lag of the dependent variable in equation i. The coefficients for the remaining lags, whether own or not, have a prior mean equal to zero.

As indicated, the prior information usually departs from the assumption of independence among the components of $\bar{\beta}_0$, i.e. from a diagonal Ω_0 matrix, whose entries of the main diagonal (ω_{hh}) will be given by one of the following prior variances:

— For the coefficients associated with the own lags:

$$\sigma^2_{ijs} = \left(\frac{\tau_2}{s^{\tau_4}}\right)\sigma^2_{\varepsilon i}; \quad i = 1, ..., n; \quad i = j; \quad s = 1, \qquad [I.17]$$
$$j = 1, ..., n$$

— For the coefficients associated with the lags of the rest of the variables:

$$\sigma^2_{ijs} = \left(\frac{\tau_2\,\tau_3}{s^{\tau_4}}\right)\left(\frac{\sigma^2_{\varepsilon i}}{\sigma^2_{\varepsilon j}}\right); \quad i = 1, ..., n; \neq \mathbf{j}; \quad s = 1, ..., \qquad [I.18]$$
$$j = 1, ..., n$$

— For the coefficients associated with the lags of the deterministic variables:

$$\sigma^2_{ijs} = \tau_2\,\tau_3\,\sigma^2_{\varepsilon i}; \quad i = 1, ..., n; \quad s \qquad [I.19]$$
$$j = n + 1, ..., n + d$$

where σ^2_{ijs} is the prior variance for the coefficient relating to lag s of variable j in equation i. Thus, for example, σ^2_{231} would be the prior variance of the first lag coefficient of the third variable of the system in the second equation.

The location of these variances in Ω_0 is given by:

$$\omega_{hh} = \sigma^2_{ijs} \qquad [I.20]$$

$$\text{where } h = \begin{cases} (i-1)\,[nm + d] + n(s-1) + j & \text{si} \quad j \leq n \\ (i-1)\,[nm + d] + nm + (j-n) & \text{si} \quad j > n \end{cases}$$

In these prior variances, τ_2 controls the overall degree of uncertainty with which the prior information is incorporated into the model estimation process; as τ_2 grows, the distribution is less informative, becoming diffuse at the limit. τ_3 controls the degree of uncertainty of the lags of other variables in relation to that of the own lags; at the limit, when τ_3 is equal to zero, the prior information defines a model made up of n AR(m) univariate processes. τ_4 controls the speed at which the variance diminishes with the lag, and τ_5 the relative uncertainty of the deterministic component. Finally, $\sigma^2_{\varepsilon_i}$ and $\sigma^2_{\varepsilon_j}$ represent the entries of the main diagonal of Σ, and are a measure of the size of the fluctuations of the variables i and j. Their role in the prior information is to allow comparison of the degree of uncertainty with the scale of the fluctuations.

Although the hyperparameterisation of Σ, possible, the usual practice has, as indicated, been to make conditional upon Σ, estimating it on the basis of the residuals arising in AR(m) univariate models estimated by OLS.

The time variation of the model remains to be characterised. This resides on the matrices S and ϕ, and its representative specification is as follows:

$$\begin{aligned} S &= \text{diag}\,(S_1, ..., S_n) \\ S_i &= \text{diag}\,(\tau_6); \quad i = 1, 2, ..., n \\ \phi &= \text{diag}\,(J_1, ..., J_n)\,\Omega_0 \\ J_i &= \text{diag}\,(\tau_7); \quad i = 1, 2, ..., n \end{aligned} \qquad [I.21]$$

where diag defines diagonal matrices by blocks whose entries of the main diagonal are those included in brackets, S_i and J_i are kxk matrices, τ_6 controls the coefficients of the first-order autoregressive process which characterises the law of motion of the coefficient vector β and τ_7 controls the degree of time variance actually incorporated into the model. Note, in particular, that τ_6 =1 and τ_7 =0 give the version of the model without time variation. Note also that time variation is proportionate to the matrix of prior variance of the vector β_0, which allows for relative evaluation of the degree of time variation.

At this point it will have become clear that the specification of the prior information described above is incomplete in the sense that it depends on an unknown hyperparameters vector τ. From a strictly Bayesian standpoint, the prior information should not contain unknown (hyper)parameters. Indeed, a strict Bayesian implementation would require the specification of distributions for hyperparameters and integration over the relevant range to obtain the posterior distribution. However, the usual practice has involved two approximative procedures in BVAR methodology applications.

The first consists of using the posterior distribution associated with the particular numerical value of the vector τ which reflects directly the empirical regularities 1) to 3) described earlier. For example:

$$\tau = \begin{bmatrix} 1 \\ 0{,}2 \\ 0{,}5 \\ 1 \\ 10^6 \\ 1 \\ 0{,}001 \end{bmatrix} \qquad [I.22]$$

This procedure was characteristic in the initial applications of the methodology, and is formally tantamount to assuming that the vector τ is a degenerate random vector with a mass of one in the specific choice [I.22].

The second procedure consists of using the posterior distribution associated with a specific numerical value selected in accordance with a goodness-of-fit criterion. Two commonly used criteria are the minimisation of a loss function defined in terms of predictive power statistics and the maximisation of the model's likelihood function.

Regarding this latter criterion, note that, given the assumption of normality, the model's likelihood is as follows:

$$\prod_{t=1}^{T} L\left(Y_t \middle| X_{t-1}, \Sigma, \tau\right) = (2\pi)^{-T/2} \prod_{t=1}^{T} \left|\Psi_{t-1}\right|^{-1/2}$$
$$\exp\left[-\frac{1}{2}\left(Y_t - X_{t-1}\beta^*_{t-1}\right)' \Psi^{-1}_{t-1}\left(Y_t - X_{t-1}\beta^*_{t-1}\right)\right] \qquad [I.23]$$

where

$$\Psi_{t-1} = X_{t-1}\,\Omega^*_{t-1}\,X'_{t-1} + \Sigma$$

The approximative criterion based on the likelihood function thus consists of maximising [I.23] with respect to τ and obtaining the posterior distribution associated with this optimum vector. The Bayesian justification of this procedure is that it can provide a reasonable approximation to the complete process of integration. Specifically, if a diffuse prior distribution is assigned to τ the posterior distribution of the coefficient vector β will be a weighted mean of the posterior distributions associated with each specific value of τ with weights given by the value of the likelihood in that specific value. Thus, by choosing the posterior distribution associated with the value of τ that maximises [I.23], we are in fact using the posterior distribution with most weight in the integration process. When the values of τ with high likelihood give rise to similar associated posterior distributions, the procedure approximates reasonably to the true posterior distribution.

To conclude this section, emphasis should be placed on the flexibility provided by the Bayesian dimension of VAR methodology, in the sense of allowing confrontation with the sample information of a broad parametral range that provides, in turn, a broad representative generality from the statistical standpoint: from the univariate AR model to the VAR model, this latter model being obtained as a particular case of the BVAR framework when the prior information selected is diffuse, i.e. when τ_2 tends to infinity. In this case, $\Omega_0^{*\,-1}$ tends to zero and, as can be clearly perceived in the scheme updating the mixed estimator presented in [I.15], the scheme for updating the prior information generates the OLS estimate of the model. The univariate AR model would be obtained by specifying diffuse prior information as in the UVAR case and, further, by making the hyperparameter that controls the degree of uncertainty of the lags of the other τ_3 variables equal to zero.

I.3.4. Efficiency of joint estimation

In the case of the UVAR model it was concluded that the single-equation estimate is efficient because all the equations have the same explanatory variables. It seems advisable to question whether this result holds for BVAR models.

The fact the Theil mixed technique has been used as an estimation method means fresh resort can be made to the SURE framework, verifying that the response to this enquiry is negative. Specifically, it is a question of checking whether, with the incorporation of the prior information, the characteristic whereby the set of explanatory variables is the same in all the equations of the system holds or not. This can be done returning to expression [I.9], which combines the prior and the sample information and in which I and X_{t-1} make up the set of explanatory variables. Generally, $I \neq X_{t-1}$, whereby the BVAR model contains two blocks of equations whose explanatory variables differ and, therefore, the result of the UVAR framework is not applicable.

In fact, the condition for the single-equation estimate to be efficient in the BVAR framework is that the prior variance of the coefficients should be a multiple of the residual variance in each of the equations (7). This condition is not met by the usual type of prior information in the applications of the BVAR methodology described in the preceding section, since in each equation the own lags are given priority, with the outcome that the variance of their coefficients is greater than that of the rest of the coefficients of the equation. Thus, generally, the joint estimation of all the model's equations is an efficiency requisite in the BVAR framework.

I.3.5. Cointegration

At no point in the description of the BVAR model has reference been made to the stationary or non-stationary nature of the stochastic process modelled. In fact, this distinction has been knowingly omitted and reflects the position that the Bayesian estimation perspective may accommodate both cases without any need for differentiated treatment. As stressed, the important thing is that the prior information should confront the sample information with a broad selection range, and this is done regardless of whether the process is stationary or not. Moreover, likelihood, the other source of information for the estimation process, is also immune to whether the process is stationary or not, in that the assumption that the overall sample density is normal does not depend on whether the process analysed is stationary or not. Therefore, from a Bayesian standpoint, there is in principle no reason to address analysis of the stationary and non-stationary series differently.

Admittedly, this stance has been criticised when the analysis unfolds in a context of non-stationary processes with unit roots and potential cointegration relationships. Specifically, Lütkepohl (1991), Clements and Mizon (1991) and Phillips (1991) have suggested that, on the basis of prior information which takes all the coefficients to be inter-independent (both in the same equation and between equations) and which assigns a mean equal to one, or close to one, to the first own lag coefficient and of zero to the rest, the Bayesian estimation of the VAR models tends to be biased towards systems made up of univariate AR models, being incapable of

(7) See Doan, Litterman and Sims (1984).

capturing the possible common stochastic trends that characterise cointegrated processes. Sims (1991a) suggested that these critiques were poorly grounded, arguing that, owing to the superconvergence property of the estimators in the presence of cointegration relationships, these aspects tend to manifest themselves with clarity, irrespective of the type of prior information used.

Alvarez and Ballabriga (1994) furnish evidence on this matter, modifying the usual prior information of BVAR models so that it explicitly incorporates the possible existence of cointegration relationships in the process analysed and performing a minor Monte Carlo experiment with a cointegrated process that allows the power of different estimation methods for capturing the long-run relationship to be considered. The results obtained sustain Sims' proposition as opposed to that of the critics, provided that the prior distribution has been selected in keeping with a goodness-of-fit criterion.

I.4. Identification of VAR models

The foregoing methodological description has not drawn on any economic argument, excepting the minimum that may be implicit in the selection of the economic variables it is sought to analyse. This may be puzzling, but it is one of the elements that helps in shaping a sort of brand image for VAR methodology: the clear differentiation between the statistical and economic aspects of the analysis, which respectively define the specification and identification stages of a VAR model.

The foregoing discussion has thus focused on the model's specification stage. Classical (UVAR) and Bayesian (BVAR) specification methods have been proposed, but in both cases the aim has been to use the statistical generality of the autoregressive representation in [I.1]-[I.3] without contaminating it with arguments of an economic nature. The result of this process is, therefore, a purely statistical model. Or, more exactly, a reduced-form model, terminology which is confined in econometrics to instruments of statistical representation lacking economic content.

Admittedly, obtaining the reduced form may be an aim in itself if what is sought is to forecast or analyse a set of correlations. However, when the aims of the analysis include matters such as the evaluation of the effectiveness of monetary policy, the reduced form is insufficient, and an intermediate step must be set up towards a statistical model in structural form that has the economic content necessary to respond to valid questions. As has been reiterated, VAR methodology has sought from the outset to be an operational alternative to traditional simultaneous-equation macroeconomic models, the main aim of which is precisely to respond to questions such as those formulated. Thus, the extra effort of obtaining a model interpretable on the basis of the reduced form is usually an unavoidable task in VAR (both UVAR and BVAR) methodology applications. This stage of the analysis is the model identification stage.

Sims' initial work and the ensuing applications soon highlighted the fact that the identification of VAR models was one of the weakest flanks of the methodological proposal. Indeed, the critical opinion that VAR models were simple reduced forms which, as such, were not valid for the quantification of economic relationships rapidly became widespread.

Actually, this criticism was not strictly true; as can be seen in the subsequent formal description, the initial applications of the methodology used a contemporaneous causal chain equivalent to a recursive simultaneous-equation structural model. Admittedly, however, a recursive structure is rarely appropriate for describing economic reality, which is why VAR models were duly susceptible to criticism in terms of their identification, though not for the absence thereof but for its dubious credibility. This is a fresh paradox if it is recalled that, in the related original motivation, the identification of simultaneous-equation models was alleged to be lacking in credibility.

Being one of the most controversial aspects of the methodology, the identification stage focused, and continues to focus, much of the academic discussion on VAR models. What may be considered a relatively satisfactory solution to the problem has arisen in the sense that current identification methods are a substantial improvement on the method used in the initial applications and are, in turn, in keeping with the broadly non-restrictive spirit of the methodology.

I.4.1. Formal description

Conceptually speaking, the identification of an econometric model is an extremely well known generic problem which concerns the model itself, not the modelling methodology. It is commonly considered as the obtaining of a structural model on the basis of its reduced form. The structural model is economically interpretable and will be identifiable if it is made up of statistically distinguishable equations which, as such, can be retrieved unequivocally drawing on the statistical variability summarised in the reduced-form model.

As stated, the problem is not resolved opting for one or another methodology; it will face all methodologies. What can actually distinguish one methodology from another is the means of tackling the problem.

Thus, traditional simultaneous-equation models manage to make their equations statistically distinguishable by the strategy of inclusion or exclusion in the various equations of variables treated as exogenous (8). Taken to unjustified extremes, this strategy provides an illusory or incredible identification according to Sims' classification (1980).

Conversely, VAR methodology does not resort to exogeneity and uses an identification strategy combining a minimum of exclusion restrictions with conditions on the probabilistic structure of the model's error component. More specifically, a VAR model is called structural when the statistical distinction of its equations is obtained through imposing a set of restrictions (not necessarily exclusion restrictions) that ensures the orthogonality of the model's error components, allowing in turn their interpretation as original sources of economic variability.

The requirement for orthogonality for the error component is not usual in traditional structural models, and reflects a deep-seated conceptual difference with respect to whether the relevant variability from the economic standpoint is «total» or «unexpected» variability. Traditional models proceed as though it were total variability which were relevant, whereby they do not insist on the orthogonality of disturbances, something unavoidable when it is wished to analyse the dynamic implications of the model, under the conviction that it is unexpected variability that is relevant.

The orthogonality requirement also explains the equivalent use commonly given in the literature on VAR models to the terms «identification of the model» and «orthogonalisation of the error component». To be more specific, we return to the reduced form in the notation of expression [I.1], which we reproduce below (9):

$$Y_t = B(L)Y_t + DZ_t + \varepsilon_t \qquad [I.24]$$

where, it will be recalled, the components of ε_t are generally correlated, with the covariance matrix equal to Σ throughout t. The identification of a VAR model can thus be considered as the obtaining as a linear combination of ε_t of a new disturbance vector whose components are orthogonal and economically interpretable. Or, in more formal terms, as the obtaining of an invertible matrix A, nxn, such that, throughout t:

$$A\varepsilon_t = \nu_t \qquad [I.25]$$

where it is intended that the components of ν_t should represent isolated sources of economic variability (fiscal or monetary, private or public, sup-

(8) Although, from a theoretical standpoint, it is usual to find identifications based on restrictions of the variance and covariance matrix, the use in practice is limited.

(9) For the sake of notational simplicity we will continue developing the outline in terms of the model whose coefficients are not time-dependent.

ply or demand, etc.), so that their variance and covariance matrix is diagonal, which, moreover, without loss of generality, can be normalised to the identity. Note that the matrix A actually provides the connection between the reduced and structural forms of the VAR model. Pre-multiplying the reduced-form VAR model [I.24] by A gives the structural VAR model:

$$AY_t = AB(L)Y_t + ADZ_t + \nu_t \qquad [I.26]$$

Or, what is equivalent:

$$C(L)Y_t = GZ_t + \nu_t \qquad [I.27]$$

where

$$C(L) = A[I - B(L)]$$

$$G = AD$$

Note that the model [I.26]-[I.27] has indeed the form of a traditional structural model, with the particularity that all the predetermined variables are lagged endogenous variables, with the exception of the deterministic component, and that the error component is orthogonal. It is these same particularities which define it as a structural VAR model.

The expressions [I.26]-[I.27] also allow concrete form to be given to the foregoing affirmation, relative to the identification of the VAR models via the combination of restrictions in the contemporaneous coefficient matrix and conditions in the probabilistic structure of the error component: it is matrix A which contains the impact coefficients, and it should be selected so as to ensure that the condition holds regarding both the orthogonality of the structural disturbances and the statistical distinction of the system's equations, so that the structure is effectively identified. A selection of A ensuring both conditions is its specification as a triangular matrix, which is known as the Choleski scheme [see, for example, Sims (1980)]. This was the usual choice in the early applications of the methodology, which, as can now be clearly appreciated, is equivalent to a contemporaneous causal chain and converts to the model [I.26]-[I.27] in a recursive structural model.

Admittedly, the recursive strategy seems to be technically correct in that it generates an orthogonal vector and a structure made up of distinguishable equations. However, as mentioned, recursive structures are not generally appropriate for describing economic reality owing to the fact that they do not incorporate the relationships of a simultaneous nature that normally characterise such reality [see, for instance, Cooley and Leroy (1985)]. They may be criticised in this respect for using barely credible contemporaneous restrictions and, as a result, they fail in the attempt to isolate sources that are credibly interpretable in economic terms, a fundamental aspect of the identification process.

The advance of VAR models in the literature on identification has moved in the direction of breaking recursiveness, considering more general specifications of the A matrix that give rise to more credible structural models. To obtain these specifications, resort has been had essentially to two types of identification restrictions: one short-run (10) [see, for example, Bernanke (1986), Blanchard and Watson (1986) or Sims (1986a)], and one long-run [see, for instance, Blanchard and Quah (1989)].

The short-term restrictions are implemented via the specification of zeros in specific positions of the A matrix, justified normally by lags in the reception of informative flows by certain economic agents. For example, the delay with which the monetary authority receives information in relation to macroeconomic developments may warrant the assumption that the interest rate does not respond contemporaneously to disturbances in output and price levels: two zeros in the A matrix that may help identify the supply of and demand for the economy's liquid stocks. The denomination of short-run with which reference is made to this type of restriction is clear, since what is involved is restricting exclusively the contemporaneous effect of certain disturbances.

The long-run restrictions are usually grounded in economic theory and, as their name indicates, they restrict the long-run effect of certain disturbances in certain variables, leaving short-term dynamics free. For example, the model may include the restriction that monetary disturbances do not have real effects in the long run: a restriction based on the principle of long-run monetary neutrality.

The use of long-term restrictions requires the use of a stationary representation (11), so that the long-run effects are well-defined; i.e. so they are not explosive. Formally, imposing these restrictions is equivalent to restricting certain linear combinations of the matrix of long-run effects associated with the representation of moving averages (MA) of the structural model. Note that under the assumption of stationarity, the polynomial C(L) in [I.27] may be inverted, giving rise to the following MA representation for the non-deterministic component of Y_t:

$$Y_t - M(L)DZ_t = M(L)A^{-1} \nu_t \qquad [I.28]$$

where

$$M(L) = [I - B(L)]^{-1} = C(L)^{-1} A$$

(10) Note that a Choleski scheme involves the use of short-run identification restrictions, though of an exclusively recursive nature.

(11) Nonetheless, the restrictions generally affect non-stationary variables.

The matrix of long-run effects of the various structural disturbances is the sum of matrices defined by the polynomial M(L) A^{-1}, each of which determines the effect of the disturbances in the different time horizons. That is to say, the matrix of long-run effects is given by:

$$M(1)A^{-1} = \sum_{i=0}^{\infty} M_i\, A^{-1} \qquad [I.29]$$

And, as stated, the long-run restrictions are equivalent to restricting certain linear combinations of the entries of the matrix in [I.29], which can be expressed as follows:

$$\underset{p \times n^2}{W} \; \underset{n^2 \times 1}{\mathrm{vec}\left[M(1)A^{-1}\right]} = \underset{p \times 1}{c} \qquad [I.30]$$

where the operator vec(•) transforms the m x n matrices into mn x 1 vectors stacking their n columns, and p represents the number of restrictions. In the particular case that cancels the long-run effect of disturbance i in variable j, the value of c will be zero and the W matrix will be 1 x n^2, with a one in the entry [(j – 1)*n] + i, i, j = 1, ..., n and zeros in the remaining entries.

Significantly, the set of short- and long-run restrictions described above shapes a framework for highly parsimonious identification from the restrictive standpoint: when the exclusion is used it is so solely with the contemporaneous impacts, without excluding potential lagged effects, and when it restricts the lagged effects it does so in a lax fashion, conditioning only the long-run effect. In this respect, we discussed earlier that the VAR identification framework observes the relatively unrestrictive spirit of the methodology.

I.4.2. Estimation of the structural model

The estimation of the structural VAR model remains to be tackled. This specifically involves the estimation of the matrix polynomial C(L) and of the G matrix of the model [I.27]. Here, use of the expressions [I.24]-[I.27] will be necessary.

Drawing on expression [I.25], which related reduced-form to structural-form disturbances, the compatibility between the variance and covariance matrices of the disturbances of the reduced- and structural-form models imposes the following relationship between Σ and the A matrix of contemporaneous effects (remember that the structural error component covariance matrix has been standardised to the identity):

$$A\Sigma A' = I \qquad [I.31]$$

Or, in an equivalent manner:

$$\Sigma = A^{-1} A^{-1'} \qquad [I.32]$$

Note below that the very expressions [I.24]-[I.25] suggest the possibility of estimating the structural model in a two-stage procedure:

Stage 1 Estimation of the D and B(L) coefficient matrices of the reduced-form VAR, and a consistent estimator for $\Sigma, \hat{\Sigma}$, based on the resulting $\hat{\varepsilon}_t$ residuals.

Stage 2: Use $\hat{\varepsilon}_t$ from stage 1 along with the conditions [I.30] and [I.32] to obtain the maximum-likelihood estimator of the A matrix.

The conjunction of the first-stage D and B(L) estimators with the second-stage A estimator A then allows the G and C(L) estimators of the structural model to be obtained.

Stage 1 does not introduce new elements, simply considering estimation of the reduced-form VAR by means of the methods described in the specification sections I.2. or I.3, depending on whether the classical or Bayesian version of the model is chosen.

Stage 2 proposes maximising, in relation to the matrix of A coefficients, the sample likelihood of the series of reduced disturbances obtained in the first stage, taking into account the possible long-run restrictions [I.30] and the compatibility condition [I.32]. To specify, note that, under the assumption of normality, the likelihood of the estimation problem of stage 2 is, taking logarithms and without taking into account the constant:

$$-\frac{T}{2}\ln|\Sigma| - \frac{1}{2}\sum_{t=1}^{T} \hat{\varepsilon}_t' \, \Sigma^{-1} \, \hat{\varepsilon}_t \qquad [I.33]$$

The estimation problem in stage 2 is hence to obtain the A matrix that maximises [I.33] subject to the conditions [I.30] and [I.32]. Note that the number of different conditions in [I.32] is not n^2 but $(n^2+n)/2$, since Σ is symmetrical. Therefore, the maximum number of contemporaneous coefficients other than zero that can be determined using the conditions in [I.30] and [I.32] is $[(n^2+n)/2]+p$; the rest of the coefficients equal zero, constituting short-term identification restrictions. Thus, when use is not made of long-term restrictions (p=0), the number of short-term restrictions (nil entries of the A matrix) should, at least, be equal to $(n^2-n)/2$.

This two-stage estimation procedure we have described is, in fact, that most used in VAR methodology applications. It is attractive for two reasons. First, it is in keeping with the idea of the clear separation of statistical specification restrictions from economic identification restrictions, a distinctive feature -as we have mentioned- of VAR methodology. And further, if the model is exactly identified, the method generates efficient estimators of the structural coefficients, since they are equivalent to those that would be obtained via the direct estimation of G and C(L) through the maximum likelihood method. The reason is that, under the assumption of normality, the information matrix on the likelihood of the structural VAR model [I.27] is diagonal with respect to [D, B(L)] and A, a condition established in Durbin (1970) to justify the efficiency of the two-stage procedure.

II

USES OF VAR MODELS

As seen in the identification section, the requirement that structural disturbances be orthogonal is a distinguishing feature of VAR methodology, the purpose of which is to isolate the primitive sources of economic variability. These are to be found in the behaviour of supply or demand, in the public or private sectors, or else in the external or domestic sectors of the economy. It should come as no surprise, therefore, that analysts who draw on VAR modelling methodology usually have a particular interest in the dynamic effects of these primitive disturbances on the course of the observable variables which characterise the economic framework it is sought to study; i.e. in the effects of ν on Y, according to our notation.

For the same reason, it should come as no surprise either that analysts are more interested in recovering the structural MA representation in [I.28] than the structural autoregressive representation in [I.26]-[I.27], since it is the MA representation which directly shows the effects of ν on Y. In fact, as will be seen below, the typical uses of VAR models are based almost entirely on obtaining and analysing the MA representation of the model.

These uses are: to calculate the model impulse response function, to decompose the variance of its forecasting error and to obtain future projections. Such uses generally serve the same purpose as the customary ones of an econometric model, namely, to contrast hypotheses and make future projections of the relationships incorporated therein. But they are marked by the emphasis on the primitive sources of variability, so that the hypotheses testing is not based on the statistical significance of certain structural coefficients, but on the overall pattern of interrelationships deployed by the model through the impulse response function and the variance decomposition; likewise, the exogenous variability which can condition future projections does not derive from the variability of the observable variables which are determined outside the model, but rather from the primitive sources of disturbance incorporated into the model.

A clarification is needed regarding the existence of the MA representation of the model, on which we have said its use is largely based. As the reader will be aware, such representation exists in the stationary case, since the autroregressive representation can be inverted. The same is not true, however, in the non-stationary case associated with the existence of unit roots, so common in empirical analysis in economics. In the latter case the autoregressive representation cannot be inverted and, accordingly, the MA representation of the model does not exist, in the sense that the succession of matrices M(L) is not convergent.

Does this mean that the analysis should be reduced to the stationary framework? To confirm that the answer is negative, let us return to representation [I.27] and consider successive substitution for the first term of that expression of Y_{t-s}, s = 1, ..., H, according to the probabilistic mechanism [I.27]. This substitution process enables the first H terms of the MA form of the model to be obtained and Yt to be expressed as the sum of two components:

$$Y_t = \sum_{s=0}^{H-1} M_s A^{-1} \nu_{t-s} + E_{t-H} Y_t \qquad [II.1]$$

Or, equally:

$$Y_t = \sum_{s=0}^{H-1} \overline{M}_s \nu_{t-s} + E_{t-H} Y_t \qquad [II.2]$$

where

$$\overline{M}_s = M_s A^{-1}; \quad s = 0, ..., H -$$

and $E_{t\text{-}H}$ denotes the expected value on the information available at t-H.

The first component in [II.2] represents the contribution to the value of Y_t of the innovations occurring between the periods t-H+1 and t, inclusive; a contribution which is determined by the sum of the first H terms of the MA form of the model. The second component is the mean projection of Y_t based on the information available in the period t-H, which, as such, depends on the vector of observable variables Y between the periods t-H and t-H-m+1 (m, remember, is the number of lags) and on the deterministic component between the periods t-H+1 and t.

The convenience of the decomposition [II.2] is that it exists for any finite H and is independent of whether or not the process analysed has unit roots. We shall use it to present the description of uses which follows.

II.1. The impulse response function

As its very name suggests, the vectoral impulse response function quantifies the effect on the n variables of the system, over a time horizon of H periods, of an isolated impulse equal to one in each of the n disturbances of the model. That is to say, the function quantifies the effect on Y_{it}, i = 1, ..., n, of the disturbance $\nu_{jt-s} = 1$, j = 1, ..., n, occurring s periods previously, s = 0, ..., H–1. The calculation of the function for the entire system therefore generates n x n series of length H.

It can immediately be appreciated that these n x n series correspond to those which make up the succession of matrices $\tilde{M}_s$, s = 0, ..., H – 1, of the first component of the expression [II.2]. To check this, consider the following succession of disturbances:

$$\begin{aligned} \nu'_{t-\bar{s}} &= (0, ..., 1_j, 0, ..., 0), \qquad 0 \le \bar{s} \le H-1 \\ \nu'_{t-s} &= 0, \qquad s \ne \bar{s} \end{aligned} \qquad \text{[II.3]}$$

That is say, the j^{th} component of $t - \bar{s}$ is given a shock of one unit in the period of ν. Note then that the result of calculating the first component of the expression [II.2] with the succession [II.3] is equal to the j^{th} column of the matrix $\tilde{M}_{\bar{s}}$, Note also, that according to the decomposition [II.2], this response must be interpreted as the deviation from the mean projection $E_{t-H}\, Y_t$ induced in the system by the specific impulse.

Thus, we conclude that the impulse response function is an instrument for assessing the dynamic effect of the different sources of variability (disturbance) included in the model, and that its computation for a time horizon H is equivalent to the computation of the first H terms of the MA form of the structural model. It should be stressed that different identifications involve alternative interpretations of the different sources of variability. Consequently, the dynamic response depends on the identification scheme employed.

II.2. The decomposition of the variance of the forecasting error

One way of assessing the relative importance of the different sources of disturbance is to analyse their contributions to the model forecasting error. The motivation for this analysis is clearly seen in the decomposition [II.2], when it is observed that its first component represents, as already mentioned, the contribution to the value of Y_t of the disturbances occurring between t-H+1 and t, and, in turn, the error of forecasting Y_t with the information available at t - H. Accordingly, the analysis of the contributions to the forecasting error in fact provides information on the relevant sources of variability at time horizon H.

The contributions are analysed by calculating the variance of the forecasting error for the horizon of interest and isolating the percentages of this variance attributable to each of the disturbances of the model; hence the name «variance decomposition» given to this exercise. More specifically, the variance of the error of forecasting Y with time horizon H is the variance of the first component of the expression [II.2], which is the following (remember that the variance of ν has been standardised as equal to one):

$$\nu ar\left[\sum_{s=0}^{H-1} \overline{M}_s \nu_{t-s}\right] = \sum_{s=0}^{H-1} \overline{M}_s \overline{M}_s' \qquad [II.4]$$

The formal exercise consists then in decomposing [II.4] into components which represent the percentage of the variance of the forecasting error associated with Y_i explained by the contribution of the component ν_j, i, j = 1, ..., n, at time horizon H. This decomposition would require additional assumptions if the entries of the vector ν were temporally or contemporaneously correlated since it would not be possible to attribute the covariances clearly. In fact, no such correlations exist, so that the decomposition does not require any further hypotheses.

Indeed, given the temporal and contemporaneous orthogonality of the entries of ν the variance of any linear combination of structural disturbances shall be the sum of the variances of each of the entries involved, so that the isolation of their contribution to the overall variance simply requires the terms associated with each disturbance to be isolated and their variances summed. In the case that concerns us, the linear combination analysed is the first component of the expression [II.2], which we reproduce here for convenience:

$$\sum_{s=0}^{H-1} \overline{M}_s \nu_{t-s} \qquad [II.5]$$

Note that the terms of [II.5] which correspond to the element ν_j between the periods t – H + 1 and t are those associated with the j^{th} columns of the matrices $\overline{M}_s$, s = 0, ..., t – H + 1. These terms can be isolated algebraically by post-multiplying the matrices $\overline{M}_s$ by the instrumental matrix R_j, all of the entries of which are equal to zero, except for (j, j), which is one. Specifically, if we call the sum of all these terms P_j, then:

$$P_j = \sum_{s=0}^{H-1} \overline{M}_s R_j \nu_{t-s}; \qquad j = 1, ..., \qquad [II.6]$$

where

$$R_j = \begin{bmatrix} 0 & \dots & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 1_{jj} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & \dots & 0 \end{bmatrix}$$

and

$$\overline{M}_s\, R_j = \begin{bmatrix} 0 & \dots & \overline{M}_s\,(1, j) & \dots & 0 \\ 0 & \dots & \overline{M}_s\,(2, j) & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \overline{M}_s\,(n, j) & \dots & 0 \end{bmatrix}$$

It is therefore clear that the forecasting error [II.5] can be expressed as the sum of the n components in [II.6]. Hence:

$$\sum_{s=0}^{H-1} \overline{M}_s\, \nu_{t-s} = \sum_{j=1}^{n} \left[\sum_{s=0}^{H-1} \overline{M}_s\, R_j\, \nu_{t-s} \right] = \sum_{j=1}^{n} P_j \qquad \text{[II.7]}$$

This expression isolates the contribution to the forecasting error of each of the n disturbance components and provides the basis for calculating their contribution to the variance of such error. Specifically, given the orthogonality of the components P_j, j = 1, ..., n, the variance of the forecasting error in [II.4] can immediately be expressed as follows:

$$\mathcal{P} \equiv \nu ar \left[\sum_{s=0}^{H-1} \overline{M}_s\, \nu_{t-s} \right] = \nu ar \left[\sum_{j=1}^{n} P_j \right] = \sum_{j=1}^{n} \nu ar P_j = \sum_{j=1}^{n} \mathcal{P}_j \qquad \text{[II.8]}$$

where

$$\nu ar\, P_j = \mathcal{P}_j; \qquad j = 1, ..., n$$

The variance of the forecasting error of Y_i with horizon H is therefore the entry $\mathcal{P}(i, i)$, and the proportion of this variance explained by the disturbance ν_j is given by the ratio $\mathcal{P}_j(i, i)/\mathcal{P}(i, i)$, i, j = 1, ..., n; a proportion which, as already argued, enables the relative importance of the different sources of variability incorporated into the model to be weighed up. Again, as in the case of the impulse response function, the contributions of the variability depend on the identification scheme used.

II.3. Future projections

Unlike the calculation of the impulse response function and the variance decomposition exercise, distinctive of the VAR methodology, the making of future projections is a common use of econometric models. The term «projection» must be understood here in the broad sense, encompassing the terms forecast and simulation. Its choice, in preference to the more usual forecast and simulation, is justified by the intention of interpreting these exercises in the general sense of projecting any aspect of the future distribution of the variables included in the model, and not only their mean dimensions. The emphasis on this matter can, indeed, be affirmed as being distinctive of the VAR framework, and we shall return to it in the next section. In this section, meanwhile, we shall concentrate on obtaining mean projections.

In the terminology of the VAR methodology we distinguish between unconditional and conditional projections. The former refer to those generated by the model with the information available in the period which defines the origin of the forecast, without any condition being imposed on the future path of the variables of the model. Conditional projections, in contrast, involve certain restrictions on the future path of some of the variables of the model; for example, on the future path of the rate of interest or wages.

In any case, we can return to expression [II.2] to specify the distinction. Written for the period $t = T + h$, $h \geq 1$, the expression is as follows:

$$Y_{T+h} = \sum_{s=0}^{h-1} \overline{M}_s \nu_{T+h-s} + E_T Y_{T+h} \qquad [II.9]$$

Let us now suppose that sample information is available to period T and that the forecasting horizon is h=H. The mean unconditional projection of Y_{T+H} with the information available at T is therefore $E_T Y_{T+H}$, in accordance with [II.9]; i.e. the result of making the forecasting errors up to horizon H equal to their mean, which is zero.

The use of [II.9] and the reduced form of the model [I.24] allows us, also, to obtain the explicit way of calculating $E_T Y_{T+H}$. Specifically, let us ask about the mean unconditional projection of Y_{T+h} for the horizon h = 1, ..., H.

According to [II.9], this projection forh = 1 es E_TY_{T+1}, and according to [I.24] it can immediately be seen that:

$$\begin{aligned} E_TY_{T+1} &= B(L)Y_{T+1} + DZ_{T+1} \\ &= B_1Y_T + B_2Y_{T-1} + ... + B_mY_{T-m+1} + DZ_{T+1} \end{aligned} \quad \text{[II.10]}$$

Likewise, in accordance with [II.9], for h = 2, a mean projection equal to E_TY_{T+2}, is obtained, which according to [I.24] and [II.10] is given by:

$$E_TY_{T+2} = B_1E_TY_{T+1} + B_2\,Y_T + ... + B_mY_{T-m+2} + DZ_{T+2} \quad \text{[II.11]}$$

Continuing the argument successively for horizons h = 3, ..., H, the following expression is obtained:

$$E_TY_{T+H} = B_1E_TY_{T+H-1} + ... + B_mE_TY_{T-m+H} + DZ_{T+H} \quad \text{[II.12]}$$

That is to say, the mean unconditional projection with the information available at T at horizon H is obtained by substituting in the reduced form of the model their own unconditional projections made with the information available at T for the lags of the variables.

For their part, conditional projections add to the information used by unconditional ones information relating to the existence of certain restrictions on the path of some of the variables of the model between the origin and the final period of the horizon of the projection; i.e. restrictions on certain components of the vectors Y_{T+1}, Y_{T+2}, ..., Y_{T+H}. In general, it is possible to restrict any linear combination of these components. But the most usual type of restriction consists in fixing the future values of some of them (e.g. fixing the path of future wages), so that the consequences of that path on the rest of the economy, according to the model, can be projected.

It is a trivial matter to impose a future path on an exogenous variable; since the variable is determined outside the model, it is simply a question of fixing its value at the desired level. However, it is not a trivial matter when the variable it is wished to fix is endogenous, which, by definition, is always the case with VAR models. Here, the variable is determined in the model, so that the restriction must necessarily be made by restricting the sources of variability incorporated in the model; i.e. in terms of the disturbances of the model. This is detected immediately in [II.9] where it can be seen clearly, given the information available at T, that to restrict Y_{T+h} is equivalent to restricting the forecasting error with horizon h. That is to say, it is equivalent to requiring the deviation between the expected value and the restricted value to be equal to:

$$\tilde{Y}_{T+h} - E_TY_{T+h} = \sum_{s=0}^{h-1} \overline{M}_s\, \nu_{T+h-s} \quad \text{[II.13]}$$

where $\tilde{Y}_{T+H}$ represents the restricted value of Y_{T+h}. More particularly, the components Y_{iT+h} can be restricted at the horizons $h = 1, ..., \bar{h}$ con $1 \leq h \leq H$, thereby imposing a future path on the i^{th} component of vector Y.

What this [II.13] shows, in any case, is that the imposition of, for example, r restrictions on the future path of the variables of a VAR model is equivalent to imposing r linear restrictions on the vectors of future disturbances of the model, something that may be expressed generally as:

$$Q\,N = q \qquad \text{[II.14]}$$

where N is an nH-dimensional vector and contains the disturbance vectors ν_{T+1}, ν_{T+2}, ..., ν_{T+H}; Q is an r x nH matrix defined in terms of the matrices $\bar{M}_s$ to incorporate restrictions of the type [II.13]; and q is an r-dimensional vector which contains the constants which define the r linear restrictions imposed.

The mean conditional projection of the disturbances between T + 1 and T + H is therefore given by the mean of the vector N conditional upon [II.14], E[N|QN = q] ; and the mean conditional projection of Y_{T+H} is immediately obtained from [II.9], with h=H and taking on both sides of the expression expectations conditional upon [II.4] and the information available at T, so that:

$$E_T\left[Y_{T+H}|QN = q\right] = E_T\, Y_{T+H} + \sum_{s=0}^{H-1} \bar{M}_s\, E\left[\nu_{T+H-s}|QN = q\right] \qquad \text{[II.15]}$$

That is to say, the mean conditional projection is the unconditional one corrected by the conditional contribution of the disturbances over the time horizon of the forecast.

As a final comment, it should be stressed that, unlike in the case of the impulse response function and the variance decomposition, future projections do not necessarily depend on the identification of the model. In fact, they are clearly independent in the unconditional case in which, as we have seen, only the statistical variable summarised in the reduced form of the model is used. They are not dependent in the conditional case either when the conditions merely restrict the forecasting error, since, according to the following equation, the forecasting error is independent of the identification:

$$\sum_{s=0}^{H-1} \bar{M}_s\, \nu_{T+H-s} = \sum_{s=0}^{H-1} M_s\, \epsilon_{T+H-s} \qquad \text{[II.16]}$$

where the relationship between ν and ε given in [I.25] and the definition of $\tilde{M}_s$ in [II.2] have been used. Only if the restriction of the forecasting error involves imposing a specific path on a component of the vector ν is the identification significant, since, in that case, it is sought to restrict the specific behaviour of an agent or economic sector, for which purpose it is necessary to have identified previously the source of the economic variability of the model associated therewith.

II.4. Measure of uncertainty

Although nothing explicit has been said in this respect, the three uses described in the foregoing sections involve calculations surrounded by uncertainty, since they are based on a stochastic model with estimated coefficients, which are, in turn, random variables. Specifically, the impulse response function and the variance decomposition depend directly on the $\tilde{M}_s$ coefficient matrices of the MA form of the model (1), as clearly shown by the calculation made with [II.3] and the expression [II.8], respectively. For their part, the future projections depend directly on the autoregressive form coefficients (D, B_s) which determine $E_T Y_{T+H}$ en [II.9], as well as the first component in the same expression, which depends, in turn, on the $\tilde{M}_s$ coefficients and the error term ν.

The immediate conclusion is that the impulse response function and the variance decomposition and also the future projections are in themselves stochastic magnitudes which can be characterised by their corresponding distributions. In the case of the first two, in fact, this permits confidence intervals to be obtained for the dynamic effects generated by the different disturbances, so that it is possible to contrast hypotheses on the effects of the different sources of economic variability.

The description of the foregoing sections has been limited to the obtaining of point estimates, which may or may not be means of these magnitudes. However, as the reader will be aware, a point estimate is hardly informative. Ideally, analysts must attempt to characterise aspects of the distribution of interest which give as precise an idea as possible of the uncertainty surrounding their calculations. With a greater or lesser degree of approximation, this characterisation is possible when the model used incorporates a complete stochastic description of all its variables -always the case with VAR models-, which we discuss below.

(1) Note that, when the true model is known, there is no uncertainty as regards the impulse response functions and the variance decomposition of the forecasting error.

We start with the impulse response function and the variance decomposition which, as we have said, are a direct function of the $\tilde{M}_s$ coefficient matrices. Remember that, according to the expression [II.2], these matrices are given by

$$\tilde{M}_s = M_s A^{-1} \qquad [II.17]$$

In turn, and according to expression [I.30], the M_s matrices which make up the M(L) matrix polynomial are defined in terms of those corresponding to the polynomial B(L) of the reduced form:

$$M(L) = [I - B(L)]^{-1} \qquad [II.18]$$

Combining expressions [II.17] and [II.18] gives the following relationship:

$$\tilde{M}(L) = [I - B(L)]^{-1} A^{-1} \qquad [II.19]$$

That is to say, the matrix polynomial with $\tilde{M}_s$ coefficient matrices depends, directly and non-linearly, on the polynomial with B_s coefficient matrices of the reduced form and matrix A with contemporaneous coefficients which determine the model identification scheme.

Expression [II.19] reveals explicitly the stochastic nature of the impulse response function and the variance decomposition, showing that their distribution depends on the distributions of the coefficients in B(L) and A. It also shows, however, that this dependence is highly non-linear, with the consequence that obtaining the mean responses and decompositions is not the same as using the mean of B(L) and A in the first term of [II.19]. In practice, the Monte Carlo methods are frequently used to obtain the distributions of responses and decompositions by means of successive draws from the distributions of B(L) and A, both known multivariate normal distributions, according to our assumptions (2). In fact, this is the usual practice to obtain confidence intervals which are normally presented in the applications of the methodology.

We shall now consider the exercise of projecting the future. As mentioned above, we interpret this exercise in the sense of projecting any aspect of the future distribution of the variables included in the model, and not only their mean values. From this standpoint, a convenient way of proceeding is to consider directly the mechanism which generates future values. Specifically, let us write the VAR model taking as reference peri-

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(2) Other possible methods for characterising the uncertainty include analytic [see, for example, Lütkepohl (1990)] and bootstrapping methods [see, for example, Samaranayake and Hasza (1988)].

od T and with the error term expressed, according to the relationship [I.27], as a function of the structural disturbances vector ν:

$$Y_{T+s} = B_1 Y_{T+s-1} + B_2 Y_{T+s-2} + \dots + B_m Y_{T+s-m} + \\ + DZ_{T+s} + A^{-1} \nu_{T+s} \qquad [II.20]$$

$$s \geq 1$$

Note then that, given the path $s = 1, 2, ..., H$, it is possible to generate realisations of the observable vectors $Y_{T+1}, ..., Y_{T+H}$ by means of draws from the distributions of B(L), D, A and the disturbance vectors $\nu_{T+1}, ..., \nu_{T+H}$ and substituting successively in [II.20], which again gives us access to use of the Monte Carlo methods as a way to characterise empirically the joint distribution of the future path of the model variables.

The usual practice is to use an approximation (3), tending not to take into account the uncertainty associated with the estimation of the coefficients, which are treated as constants. Unconditional projections may therefore be made by means of successive draws from the distribution of vector N defined in [II.14], which is, on our assumptions, a multivariate normal distribution with zero mean and a covariance matrix equal to the identity matrix. Likewise, the conditional projections are obtained by means of draws from the distribution $[N|QN = q]$.

$$[N|QN = q] \sim N\,[Q' (QQ')^{-1} q, I - Q' (QQ')^{-1} Q] \qquad [II.21]$$

In either case, conditional or unconditional, analysts are able not only to characterise the mean values of the future distribution empirically, but also to accompany them with confidence intervals and, more generally, calculate the probability of any event associated with the future path of the variables included in the model, which is fundamental when the high degree of uncertainty surrounding the future path of the economy is recognised.

(3) However, from an analytical point of view there are results which contemplate the uncertainty associated with the estimation of the coefficients [see, for example, Samaranayake and Hasza (1988)].

PART TWO

A MACROECONOMETRIC MODEL FOR THE SPANISH ECONOMY

INTRODUCTION

The first part of this paper was devoted to a detailed theoretical explanation of an econometric methodology -VAR modelling- used more and more frequently in empirical work. Part two describes a macroeconometric model used periodically by the Banco de España Research Department to forecast the main magnitudes of the Spanish economy, as well as to perform simulations. Continued use of the model and the perspective offered by the time that has elapsed since it was designed have enabled the limitations of the initial specifications to be identified and its potential as a tool of monetary policy to be developed. These advances have gradually been published [Álvarez, Ballabriga and Jareño (1995), Álvarez, Ballabriga and Jareño (1997), and Álvarez, Ballabriga and Jareño (1998)] and are now extended in this paper.

When designing a model it should first be recognised that forecasting is an activity replete with difficulty. This is especially true in the context of the social sciences, for at least three reasons: 1) the factors explaining the phenomena to be predicted tend to be numerous; 2) the relationship between such phenomena and their determinants is usually complex, so that it is not known precisely; and 3) perhaps most importantly, the future path of such factors is surrounded by a high degree of uncertainty. In sum, the complexity of social reality makes forecasting inherently difficult, and this difficulty is reflected in the high degree of uncertainty normally involved.

Among the social sciences economics is, of course, no exception. Although sometimes extreme positions are held which deny, owing to its inaccuracy, that economic forecasting is of any use whatsoever, it nonetheless seems undeniable that any decision-making process in a context of uncertainty requires, to some extent, consideration of the future path of certain magnitudes. Thus, despite all the attendant risks and difficulties, economic forecasting is perceived as necessary. In particular, forecasts of the main macroeconomic magnitudes are of great interest to economic policymakers as they can indicate the desirability of changing certain elements of the policies being pursued.

Although not always consciously, economic forecasts are always explicitly or implicitly based on a model. This model may be formally represented, as in the case of the various types of econometric models. The procedure used is then transparent and may be applied as new information is received on the state of the economy. Also the model may be reproduced by persons other than those who developed it. The alternative, traditionally used by numerous analysts, are models lacking a formal representation and incorporating a considerable amount of subjective perception, in the hope – not always realised – that this will help to give them greater predictive power than other more formal models.

Any forecast of the future path of a macroeconomic variable involves a set of hypotheses which introduce a far from negligible degree of uncertainty. Accordingly, it is important that the various assumptions involved are made explicit. If these hypotheses are also accompanied by a formal description of the associated risks, then it is possible to characterise the probability of the future path of the economy.

The logical approach, given the uncertainty associated with economic forecasting, is to attempt a meaningful characterisation of such uncertainty rather than failing to take it into consideration and giving a false impression of rigour and precision. Paradoxically, controversy often arises over differences of a tenth of a percentage point between various forecasts, while the fact that our ignorance allows us to do no more than specify an interval or range within which, with a certain probability, the macroeconomic magnitude of interest will lie goes unrecognised. In this respect, econometric models in which all the variables are determined within the model itself do in fact permit the uncertainty inherent in the projections to be assessed. This is a fundamental advantage over econometric models in which certain explanatory variables are taken as given, as well as over subjective forecasts.

For the Spanish economy, most of the macroeconomic forecasts published at intervals of less than a year are based either on univariate time series models or on expert predictions, producing a void as regards forecasts derived from econometric models that capture the interrelationships between economic variables and provide both objective measurements of the uncertainty surrounding forecasts and reliable quantifications of the probability of occurrence of certain events. This void can be filled by building multivariate econometric models, such as the one dealt with in this paper.

Although the forecasting of macroeconomic magnitudes is, in any case, of great interest, following the change in the the monetary policy arrangements brought about by the approval of the Law of Autonomy of the Banco de España and the consequent setting, by the monetary authorities, of direct inflation targets, the analysis and prediction of prices has become even more important from the standpoint of the central bank. Consequently, meaningful inflation forecasts and measurements of the uncertainty associated with them have become essential, and the motivation to develop tools to make them has increased sharply. In this respect, multivariate econometric models, such as the one set out below, constitute very useful tools, which effectively supplement the range of instruments for analysing and forecasting inflation.

After this introduction, part two is structured as follows: chapter III explains the variables used in the model, chapter IV details the model specification process, chapter V indicates the main interactions existing between the different variables and chapter VI explains certain applications of the model.

III

THE MODEL VARIABLES

In general, the first critical decision to be made when building econometric models which seek to encompass the fundamental features of an economy usually relates to the choice of variables. In economic theory a large number of variables may be relevant for characterising an economy. However, in a model, the consideration of an excessive number of variables usually makes the estimates unreliable. This problem, which afflicts econometric modelling in general, is aggravated in the case of Spain by the fact that only a short run of statistical data is available. The quarterly National Accounts time series began in 1970, while many of the monetary series begin in 1974. The sample periods analysed by models including both types of variables must therefore begin no earlier than in 1974.

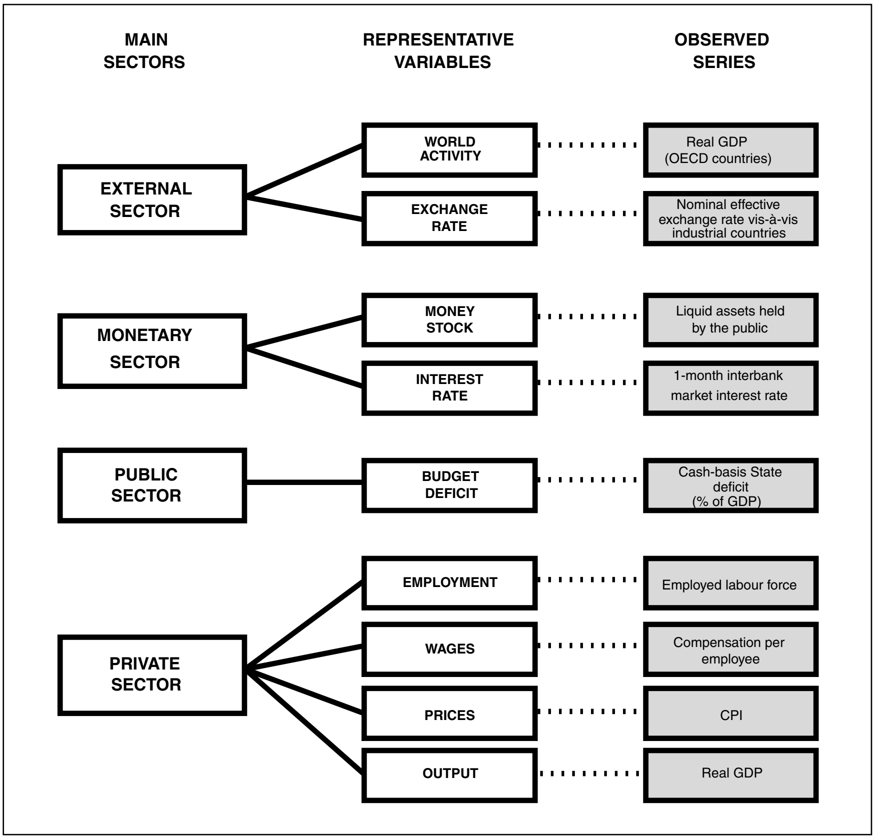
In short, the problem is how to obtain as general a picture of the Spanish economy as possible, while accepting that the number of variables used to characterise it cannot be very large. With these premises it seems appropriate to begin by considering which are the sectors of interest on which a model of the Spanish economy should be structured. Once this sectorisation has been accomplished it is necessary to determine which minimum set of variables characterises each sector. The aim of this approach is to ensure that the set of variables chosen is both restricted and capable of characterising the economy as a whole. The final step in this process is to select the available statistical series that best approximate the variables chosen. In keeping with this approach, the sectorisation of the economy used in this paper (see Chart III.1) distinguishes between the external, monetary, public and (non-monetary) private sectors, thus providing a complete, structured description of the Spanish economy.

III.1. The external sector

This sector captures the influence of the decisions of economic agents who do not belong to the Spanish economy. The opening up of

CHART III.1

SELECTION OF VARIABLES



the economy to external markets has accelerated in recent decades and given rise to a considerable increase in the interrelation between the domestic and international variables. It therefore seems relevant to include in the model some variables explicitly reflecting the external environment. Given that one of the main channels for relations between different economies is trade it seems appropriate to select the variables in accordance with its main determinants: competitiveness and external activity (1). The model will therefore include an exchange rate and a measure of activity in the rest of the world.

Generally speaking, the exchange rate is included for two reasons. First it is a variable which conditions monetary policy. From this stand-

(1) Naturally, the demand for imported goods and services also depends on the country's level of activity. Nonetheless, given its domestic nature, this variable is included in the non-monetary private sector.

point, a good approximation for the initial years of the sample would be the peseta/US dollar rate, while for the last few years of the period analysed the peseta/D-Mark rate would be more suitable. Second, the exchange rate is the transmitter of external effects on the economy's purchasing power. Thus the exchange rate will be indicative of the competitiveness of the national economy. If a perspective based on competitiveness is adopted, as in this study, it would be more appropriate to use a multilateral exchange rate than a bilateral one, because the appreciations or depreciations of the national currency against the currencies of some trading partners are generally offset by its depreciations or appreciations against the currencies of other countries. Of the multilateral exchange rates regularly published the one selected for this model is the nominal effective exchange rate vis-à-vis industrial countries (E).

A variable of activity in the rest of the world ought, in theory, to include all the other countries in the world; however, the quality of the statistics on numerous countries is not fully satisfactory, so it may be advisable to restrict the geographic scope to highly developed countries, since these provide the best quality information. The empirical evidence for the Spanish economy seems to suggest that the OECD is the most appropriate group of countries, both because of its high share in world output and because of the reliability of the statistics of its members. Consequently, the series used to represent world activity is that of the real gross domestic product of the OECD countries (GDP*).

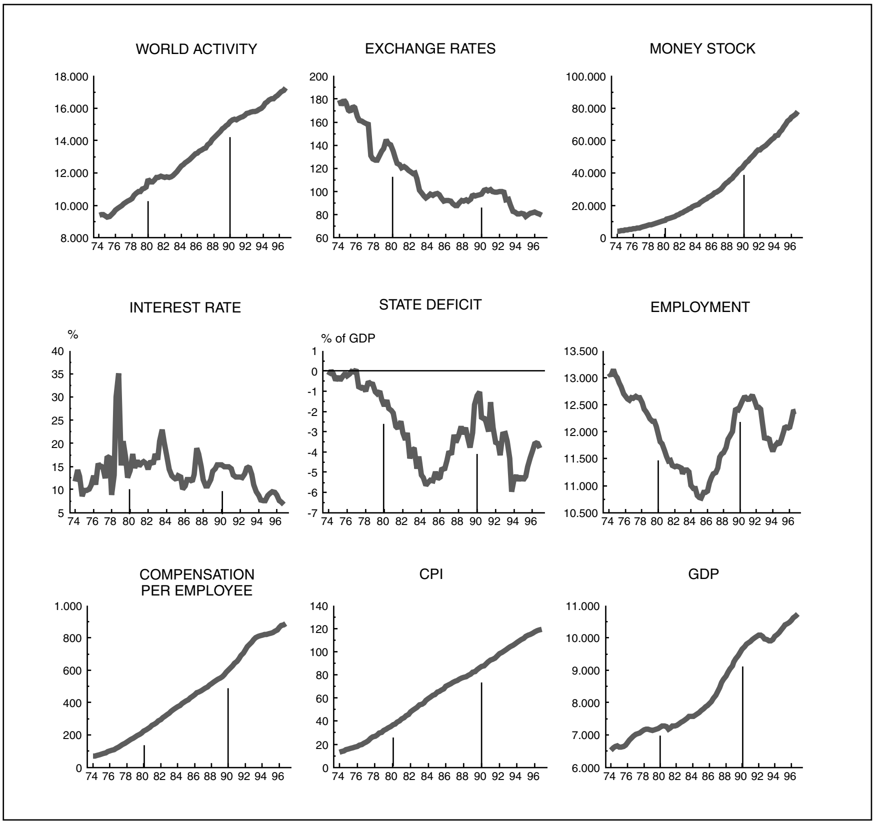
III.2. The monetary sector

This sector is associated with the actions of the monetary authority and the financial institutions. The behaviour of this sector can be characterised by a price and a quantity variable: the interest rate and the money stock.

The interest rate is the preferred instrument for implementing monetary policy, insofar as it is a determinant of the consumption and investment decisions of economic agents. Although numerous interest rates exist -both real and nominal, short- and long-term-, and each has a different impact on the economy, for simplicity's sake a single interest rate is used in this model to characterise the stance of monetary policy and its effect on the spending decisions of economic agents. The available evidence suggests that interbank market interest rates can adequately fulfil this role. Accordingly, the series selected is the «interbank market one-month non-transferrable deposit interest rate» (I).

Consideration of the money stock variable is prompted by the fact that, although the demand for money equations estimated in recent years

SERIES USED IN THE MODEL: LEVELS



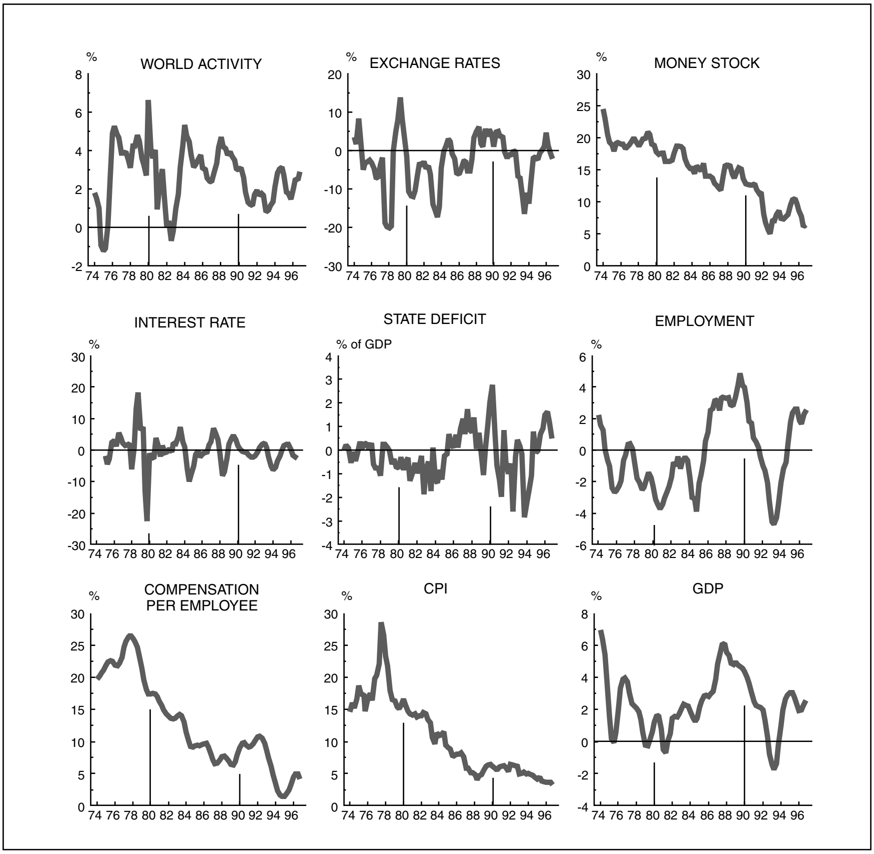
Sources: Banco de España, Instituto Nacional de Estadística and OECD.

have proven unstable, the money stock was the intermediate target of monetary policy until 1994 and is currently used as an indicator in monetary programming. The aggregate «liquid assets held by the public» (M) seems an appropriate choice here; it was used as an intermediate target in the central part of the sample period and it is an indicator of the monetary pressures in the economy.

III.3. The public sector

The complexity and diversity of public sector activity can be approximated by its budgetary aspect, which can in turn be represented by the budget deficit. Despite the limitations involved in reducing this sector to a single variable, this decision has the advantage of helping to keep the size of the model within manageable limits.

SERIES USED IN THE MODEL: YEAR-ON-YEAR RATES (a)



Sources: Banco de España, Instituto Nacional de Estadística and OECD.

(a) In the case of the interest rate and the State deficit, year-on-year changes are used.

The series selected is the State Cash-Basis Deficit (2) (D), since it records payments, receipts and non-financial operations irrespective of the way in which the State records its operations. All the same, given that a large part of the variability of this series is a result of administrative factors, which should not have economic effects (3), a moving average of four terms is considered. Also, as is customary, the series is expressed as a percentage of nominal GDP. It should be pointed out that the reason

(2) The series used is estimated at the Banco de España. It differs from the (non-financial) cash-basis deficit according to the National Audit Office in that it corrects certain time lags in some of the spending items.

(3) For example, changes to tax collection timetables should not affect agents' perceptions of the state of public finances.

TABLE III.1

SERIES USED IN THE MODEL: SOURCES

Sector	*Variable (Abbreviation)*	*Series*	*Source*
External	World activity (GDP*)	GDP at constant prices of the OECD countries.	OECD
	Exchange rate (E)	Nominal effective exchange rate vis-à-vis industrial countries. Index 1990 = 100. Average of monthly data.	Banco de España
Public	Public Deficit (D)	State cash-basis deficit according to the Banco de España. Cumulative total of monthly data. Series couverted using non-centred four-term moving averages, expressed as so many basis points of nominal GDP.	Banco de España
Monetary	Interest rate (I)	Interest rate on one-month non-transferable deposits in the interbank market. Average of monthly data.	Banco de España
	Money Stock (M)	Liquid assets held by the public. Average of monthly data. Millions of pesetas.	Banco de España
Private	Prices (CPI)	Consumer Price Index. Index 1992 = 100. Average of monthly data.	Instituto Nacional de Estadística and, Matea, Briones and Regil (1995)
	Wages (W)	Compensation per employee in national accounting terms. Thousands of pesetas.	Banco de España
	Level of activity (GDP)	Gross Domestic Product at constant prices. Base 1986. Billions of pesetas.	Instituto Nacional de Estadística
	Employment (L)	Employment labour force according to the Labour Force Survey. Thousands.	Instituto Nacional de Estadística, Perea y Gómez (1994), and Artola, García Perea and Gómez (1997)

for not using a series reflecting the whole of general government, as would be desirable, is simply the considerable lag with which information is obtained on general government other than the state.

III.4. Private sector (non-monetary)

The purpose of this sector is to represent the decisions of domestic agents in goods and services markets, as well as in the labour market. In view of such purpose, price and wage and output and employment levels have been selected (4).

Inclusion of the price variable is justified for at least two reasons: 1) it is an important reference variable in the decision-making of private economic agents, and 2) it directly reflects the national economy's inflationary situation, control of which is the monetary authority's highest priorities. The series chosen to represent prices is the consumer price index (CPI) (5), as it is the series to which private economic agents usually refer and in terms of which the central bank sets its targets. The choice of alternative series, such as National Accounts deflators, has been ruled out owing to the greater delay in the receipt of the data, the frequency and magnitude of revisions, and, basically, the lower attention they receive from the various economic agents.

The wage variable reflects, in part, the terms on which equilibrium is established in the labour market, and also indicates the possible existence of nominal pressures on the path of prices. If the aim is to capture precisely the price formation process, then labour costs, which are an important component of firms' variable costs, should be stressed. Compensation per employee (W), which includes wages and social security contributions, payable by both the employer and the employee, is the most appropriate variable to represent the cost of labour.

Finally, output and employment have been selected as the variables to reflect the level of real activity in the economy. The specific series chosen are gross domestic product (GDP) and the employed population (L).

The level and twelve-month growth rate of the variables included in this model are shown in Charts III.2 and III.3 for the whole sample period considered. The source of the same is given in Table III.1.

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(4) Despite the private sector label, the variables in this group refer to the economy as a whole.

(5) In fact, the series used in this model has been corrected for the effects of changes to VAT rates in the first quarter of 1995. This correction has been made by estimating the effect of the changes on the level of the series, and removing it from this and all subsequent quarters.

IV

THE SPECIFICATION OF THE MODEL

IV.1. Description of the structure of the model

As indicated in section I.3.3, the specification of the prior information incorporated into BVAR models is usually based on the empirical regularities observed in the behaviour of the economic series, which are introduced by means of a set of hyperparameters such as that specified in [I.22]. In any case, these regularities should be considered a set of minimum properties common to a large number of economic series, so that the practical application of BVAR models should not solely be limited to considering these regularities as the whole of the prior information set. The specification of prior information in accordance with such regularities is a point of departure for specifying a wider set of prior information which will depend on the data used and the problem under study. In an attempt to accommodate the particularities existing in the Spanish economy, the prior information set used in the model specified has the following features:

— *Prior distribution of the coefficients*

It is assumed that the prior distribution of the model coefficients is multivariate normal. In formal terms, if β denotes the column vector which includes all the model coefficients (1), then:

$$\beta \sim N_{nm+d}[\bar{\beta}(\tau), \Omega(\tau)] \qquad [IV.1]$$

n being the number of endogenous variables in the system, m the number of lags in the model and d the number of deterministic variables. To characterise this distribution completely it is necessary to specify the vector of means β and the matrix of variances and covariances Ω which are

(1) In order to simplify the notation the time subscript is omitted from $\bar{\beta}_0$ and Ω_0.

a function of a small-dimensional vector of hyperparameters τ the entries of which appear in Table IV.1. The functional relationships between the prior means and variances and the entries of the vector of hyperparameters τ are specified below.

Following the usual practice in the literature, it is assumed that the matrix of variances and covariances is diagonal, so that the model coefficients are a priori independent, i.e. for each coefficient:

$$\beta_{ijs} \sim N[\bar{\beta}_{ijs}(\tau), \sigma^2_{ijs}(\tau)] \qquad \begin{matrix} i = 1, ..., n \\ j = 1, ..., n + d \\ s = 1, ..., m \end{matrix} \qquad \text{[IV.2]}$$

i being the number of the equation, j the number of the explanatory variable (both for the n stochastic variables of the system and for the d deterministic variables) and s the number of the lag. In short, to fully characterise the distribution of each coefficient it is only necessary to specify its prior mean and variance.

The functional relationships between the prior means and variances and the entries of the vector of hyperparameters are detailed below.

— *Prior mean of the coefficients of the stochastic variables*

The model distinguishes between two groups of variables with different means associated with the first own lag, τ_0 and τ_1. The coefficients associated with the rest of the lags have a zero mean. In the form of an equation:

If $i \in C_1$

$$\bar{\beta}_{ijs} = \begin{cases} \tau_0 & i = j, \quad s = 1 \\ 0 & \text{otherwise} \end{cases} \qquad \text{[IV.3]}$$

If $i \in C_2$

$$\bar{\beta}_{ijs} = \begin{cases} \tau_1 & i = j, \quad s = 1 \\ 0 & \text{otherwise} \end{cases} \qquad \text{[IV.4]}$$

where C_1 refers to the set of world activity, money stock, compensation per employee, price, output and employment variables; and C_2 refers to the set of exchange rate, interest rate and budget deficit variables.

TABLE IV.1

HYPERPARAMETERS ON WHICH THE PRIOR MEAN AND VARIANCE OF THE MODEL COEFICIENTES DEPEND

τ_0: Prior mean of the coefficient of the first own lag for a group of variables.

τ_1: Prior mean of the coefficient of the first own lag for the rest of the variables.

τ_2: Overall degree of uncertainty.
— This hyperparameter determines the relative weight of prior information.

τ_3: Relative uncertainty of the rest of the variables.
— This hyperparameter indicates the importance of the rest of the variables.

τ_4: Relative uncertainty of the lags.
— This hyperparameter indicates the extent to which the lags closer in time have greater informative content than more distant in time.

τ_5: Relative uncertainty of the constant term.
— This hyperparameter indicates the uncertainty as to the value the constant may take.

τ_6: Relative uncertainty of the seasonal dummy variables.
— The hyperparameter shows the uncertainty as to the value the coeficients associated with the seasonal variables may take.

τ_7: Variation of the coefficients over time.
— This hyperparameter controls the variance of the random walk process for each coefficient.

τ_8: Relative uncertainty of the domestic variables in the world activity equation.
— This hyperparameter enables the world activity variable to be specified as exogenous to the rest of the system when it takes a value of zero.

τ_9: Relative uncertainty of world activity in the rest of the system.
— This hyperparameter makes it possible to distinguish from the rest of the variables, the prior uncertainty associated with the coefficients of world activity in the rest of the variables.

— *Prior mean of the coefficients of the deterministic variables*

A zero prior mean is specified for this type of variable:

$$\bar{\beta}_{ijs} = 0 \qquad \begin{array}{l} i = 1, ..., n \\ j = n + 1, ..., n + d \\ s = 0 \end{array} \qquad \text{[IV.5]}$$

where s = 0, since it is assumed that the deterministic variables only have a contemporaneous effect.

— Prior variance of the coefficients of the own lags

Those lags corresponding, in each equation, to the variable which appears in the first term are called own lags. Their prior variance is considered to be determined by:

$$\sigma^2_{ijs} = \frac{\tau_2}{s^{\tau_4}} \sigma^2_{\varepsilon_i} \qquad i = j \quad i = 1, \ldots \qquad s = 1, \ldots, m \qquad [IV.6]$$

As can be seen, the variance depends on the hyperparameters τ_2 and τ_4 and the element $\sigma^2_{\varepsilon_i}$. τ_2 is a global hyperparameter on which all the prior variances of the system depend. This hyperparameter determines the relative weight of prior information. Thus, a zero value means no account is taken of sample information, while an infinite value means no account is taken of prior information. The hyperparameter τ_4 indicates the extent to which lags closer in time have a greater information content than lags more distant in time. High values of this parameter thus indicate that the distant coefficients are, a priori, less important, while their importance will be greater if the value is low. Finally,$\sigma^2_{\varepsilon_i}$ is obtained, following Litterman (1986), as the residual variance of an AR(m) model with a constant term.

— Prior variance of the coefficients of the lags of the other variables

The prior variance of the coefficients of the variables which, in each equation, do not appear in the first term is specified as:

$$\sigma^2_{ijs} = \frac{\tau_2 \tau_3}{s^{\tau_4}} \frac{\sigma^2_{\varepsilon_i}}{\sigma^2_{\varepsilon_j}} \qquad i \neq j \quad i = 1, \ldots, n \qquad j = 1, \ldots, n \qquad s = 1, \ldots, r \qquad [IV.7]$$

This variance depends on, apart from the terms commented on above, a further hyperparameter τ_3 The effect of this hyperparameter is to indicate the importance of the lags of the other variables. A low value implies scant interaction between variables, while a high value means that the interactions are considerable.

— *Prior variance of the constant term*

The prior variance of the constant term depends on the hyperparameter τ_5. A high value of τ_5 implies that hardly any prior information is available on the value the constant may take, and a value of zero implies that the knowledge is complete. A zero τ_5 together with a zero prior mean is equivalent to not including a constant term in the model.

$$\sigma^2_{ijs} = \tau_2 \tau_5 \sigma^2_{\varepsilon i} \qquad \begin{array}{l} i = 1, ..., n \\ j = n + 1 \\ s = 0 \end{array} \tag{IV.8}$$

— *Prior variance of the coefficients of the seasonal dummies*

Since the model includes variables that exhibit seasonal behaviour (i.e. the consumer price index, liquid assets held by the public and employment), seasonal dummies are included in the equations. Their prior variance depends on hyperparameter τ_6. A high value of τ_6 indicates a high degree of uncertainty as to the value the coefficients associated with these seasonal variables may take, while a zero value means that the prior knowledge is perfect. As in the case of the constant, a zero value for the hyperparameter τ_6 together with a zero prior mean, is equivalent to excluding the seasonal dummies. The functional form of the prior variance shall be established as:

$$\sigma^2_{ijs} = \tau_2 \tau_6 \sigma^2_{\varepsilon i} \, I_i \qquad \begin{array}{l} i = 1, ..., n \\ j = n + 2, n + 3, n \cdot \\ s = 0 \end{array} \tag{IV.9}$$

where I_i is 1 if the variable i exhibits seasonal behaviour (as in the case of the money stock, prices and employment), and otherwise 0.

— *Time variation of the coefficients*

This model allows for the possibility that the coefficients vary over time. Specifically, each coefficient follows a random walk the variance of which is given by the hyperparameter τ_7. Obviously, if τ_7 is equal to zero, the model considered does not vary over time. The formal representation of this characteristic shall be:

$$\beta_t = \beta_{t-1} + u_t \qquad [IV.10]$$

$$u_t \sim N(0, \tau_7 I) \qquad [IV.11]$$

— *Prior variance of the coefficients of world activity in the rest of the equations and of the coefficients of the rest of the variables in the world activity equation*

The prior information under consideration implicitly assumes that all of the variables are endogenous. However, in small economies such as that of Spain, it is more appropriate to consider the possibility that world activity is exogenous; i.e. that it is not affected by the domestic variables. To attain this objective, two additional hyperparameters are introduced. The first, τ_8, captures the relative uncertainty of the domestic variables in the world activity equation, which is the first in the system. Exogeneity is obtained if τ_8 is zero. Hyperparameter τ_9, on the other hand, can be used to control the relative uncertainty of world activity in the rest of the system.

Thus, the prior variance of the coefficients of the rest of the variables in the world activity equation shall be:

$$\sigma^2_{ijs} = \frac{\tau_2 \tau_3 \tau_8}{s^{\tau_4}} \frac{\sigma^2_{\varepsilon_i}}{\sigma^2_{\varepsilon_j}} \qquad \begin{array}{l} i = 1 \\ j = 2, ..., n \\ s = 1, ..., \end{array} \qquad [IV.12]$$

while the prior variance of the coefficients of world activity in the rest of the equations shall be:

$$\sigma^2_{ijs} = \frac{\tau_2 \tau_3 \tau_9}{s^{\tau_4}} \frac{\sigma^2_{\varepsilon_i}}{\sigma^2_{\varepsilon_j}} \qquad \begin{array}{l} i = 2, ..., n \\ j = 1 \\ s = 1, ..., \end{array} \qquad [IV.13]$$

— *Prior variance of the coefficients of the interest rate equation*

Like world activity, the interest rate is treated differently from the other variables. Thus, in this model the interest rate is deemed to follow an AR[1] process exogenous to the rest of the system variables:

$$\sigma^2_{ijs} = \begin{cases} \tau_2 \sigma^2_{\varepsilon_i} & \begin{matrix} i = j = 4 \\ s = 1 \end{matrix} \\ 0 & \text{otherwise} \end{cases} \qquad [IV.14]$$

where the interest rate equation is the fourth in the system.

— *Prior variance of the coefficients of other deterministic variables*

In the prices equation there are also two step-type deterministic variables, to capture the introduction of VAT in Spain in the first quarter of 1986 and the change in VAT rates in the first quarter of 1992, respectively. The prior variance for these variables shall be:

$$\sigma^2_{ijs} = \tau_2 \tau_5 \sigma^2_{\varepsilon_i} \quad \begin{matrix} i = 7 \\ j = n + 5,\ n + \\ s = 0 \end{matrix} \qquad [IV.15]$$

where the prices equation is the seventh in the system.

The GDP and employment equations contain, as further deterministic variables, a broken trend with a break point in the first quarter of 1985. It should be noted that the use of deterministic trends, especially towards the end of the sample period, usually creates serious problems when it comes to forecasting, as it makes the forecasts inflexible when new information is incorporated. However, this is not true in models, like the one presented here, with Bayesian schemes for updating the coefficients, which allow for the adaptation of forecasts in the light of new information. The prior variance for this broken trend shall be:

$$\sigma^2_{ijs} = \tau_2 \tau_5 \sigma^2_{\varepsilon_i} \quad \begin{matrix} i = 8, 9 \\ j = n + \\ s = 0 \end{matrix} \qquad [IV.16]$$

the GDP equation being the eighth and the employment one the ninth.

As can be seen, the prior variances of the constant and other deterministic variables (with the exception of the seasonal ones) exhibit the same functional form. This fact gives substance to the assumption, adopted in this model, that the same degree of knowledge exists as to the values their coefficients may take, so that they are assigned the same uncertainty.

Once the prior distribution considered in the model has been established, the next step must be to estimate the reduced form thereof, combining the prior information with the sample information.

IV.2. Estimation of the reduced forms

Most of the series considered in this analysis can be characterised as non-stationary processes. The traditional solution to avoid the possibility of spurious regressions consisted in the estimation of models in differences. However, with the development of cointegration theory it has been shown that this way of proceeding is incorrect, since it involves disregarding information on the long-run relationships that presumably exist between these series, giving rise to biases in the estimated parameters. Likewise, in view of the debate which usually arises in relation to the specific number of cointegration relationships and the practical difficulties of interpreting them when the number of variables modelled is not small, one way of proceeding, used more and more frequently, is the unrestricted estimation of VAR models in levels. This procedure enables consistent estimators to be obtained, which are asymptotically equivalent to those obtained using maximum likelihood (2). Also, the consistency of the estimators is not affected by the introduction of the prior information (3) . Consequently, in the different models that have been estimated the differences have not be taken of the variables.

In the specification of multi-equation models there are generally efficiency gains when all the equations are estimated together, instead of each equation being estimated separately. As mentioned in the first part of this paper, in the case of unrestricted VAR models these efficiency gains disappear, since each equation includes the same explanatory variables. However, when the classical estimation methods are abandoned it should be taken into account that the fact that the same explanatory variables are included in each equation does not necessarily mean that the possibility of increasing the efficiency of the estimation with respect to single equation models has to be ruled out. Indeed, the condition for there not to be any efficiency gains in the estimation of BVAR models is that the covariance matrices of the prior distribution of the coefficients are

(2) See Sims, Stock and Watson (1990), and Park and Phillips (1989).

(3) See Sims (1991) and the evidence provided by Álvarez and Ballabriga (1994).

a multiple of the residual variance for each of the equations (4). This would indicate the appropriateness of multi-equation estimation for this type of model. However, as the preliminary results obtained with joint estimation of the whole system did not reveal any major differences with respect to equation-by-equation estimation, and in view of the high computational costs in a model of this size (5), following the usual practice in the literature (6) single equation estimations have been used.

Estimation of a reduced form can generally be carried out in various ways. The usual methods place a high value on considerations of unbiasedness, consistency and efficiency, criteria which would lead to the use of ordinary least square estimations when this perspective is adopted. If a pure Bayesian perspective is adopted instead, as discussed in section I.3, the point of departure should not be a prior distribution of the coefficients which depends on a small set of unknown τ hyperparameters; rather a prior distribution would also have to be associated with these parameters and the relevant integration process carried out to obtain the posterior distribution of the model coefficients. To avoid this process, which is costly, alternative procedures have frequently been used in the literature: 1) prior distributions associated with specific vectors of hyperparameters which reflect some empirical rules regarding the behaviour of the economic time series (7); 2) the prior associated with the vector of hyperparameters which maximises the likelihood of the system. From a Bayesian perspective, this approach involves approximating the mean of the a posteriori distribution through the mode. If the prior distribution of the vector of hyperparameters is uniform, this approximation will be good, provided that, for those vectors of hyperparameters for which the likelihood is high, the related posterior distribution does not differ substantially from that associated with the maximum-likelihood vector. Also, as an alternative to these criteria, when forecasting models are built, frequently some measure of forecasting error is minimised. Specifically, the criterion followed in this paper involves minimising the root of the mean square out-of-sample forecasting error (8) one to four periods ahead (9). This one-year forecasting statistic shall be called FE1.

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(4) See Doan, Litterman and Sims (1984).

(5) The proportion, in computer time, of equation by equation as opposed to joint estimation is approximately 1 to 14,000.

(6) See, for example, Sims (1989).

(7) An interpretation of this approach could be that the prior distribution of the vector of hyperparameters τ is a degenerate distribution which places the whole mass of probability at this point.

(8) The model is re-estimated with information to t and is used to predict t+s, s=1,...,4.

(9) The statistic used in this paper averages the roots of the mean square errors of the different variables for the different forecasting horizons. To avoid the criterion excessively penalizing the equations with high variability, the root of the mean square error of each equation is divided by the residual standard deviation of an AR(m) model.

TABLE IV.2

HYPERPARAMETERS ASSOCIATED WITH THE REDUCED FORMS (a)

Hiperparameters	*Models*			
	BVAR	*UVAR*	*BAR*	*MIN*
τ_0: Prior mean of the first lag of the dependent variable for the first group of variables {GDP* M, W, CPI, GDP and L}	0.921	1.0	0.921	1.0
τ_1: Prior mean of the first lag of the dependent variable for the second group of variables {E, I, D}	0.632	1.0	0.632	1.0
τ_2: Overall degree of uncertainty	0.58×10^{-2}	1.0×10^{8}	0.58×10^{-2}	0.2
τ_3: Relative uncertainty of other variables' lags	0.0476	0.5	0.0	0.5
τ_4: Relative uncertainty of the lags	1.688	1.0	1.688	1.0
τ_5: Relative uncertainty of the constant term	9×10^{6}	5.0	9×10^{6}	5.0
τ_6: Relative uncertainty of the seasonal dummy variables	81×10^{9}	5.0	81×10^{9}	5.0
τ_7: Variation of the coefficients over time	0.103×10^{-5}	0.0	0.103×10^{-5}	0.0
τ_8: Relative uncertainty of the domestic variables in the world activity equation	0.0	0.0	0.0	0.0
τ_9: Relative uncertainty of world activity in the rest of the system	1.0	1.0	1.0	1.0

Source: Authors' calculations.

(a) The set of seasonal variables is made up of (M, CPI, L). The set of non-seasonal variables is made up of {GDP*, E, I, D, W, GDP}.

Note that the specification discussed in section IV.1 is sufficiently general to encompass the various possibilities discussed. Thus, the estimation of a UVAR model within this framework is possible, as discussed in the first part, making τ_2 tend to infinity. For its part, the prior distribution which reflects the empirical rules described in section 1.3.3., on the behaviour of economic time series, is sometimes associated with the University of Minnesota, for which reason we shall refer to it as MIN. The values of the hyperparameters of this distribution appear in Table IV.2 (MIN column). Finally, the vector of hyperparameters associated with the one-year forecasting statistic is also contained in that Table (10) (BVAR column). To determine this vector, the non-standard optimisation routine described in Sims (1986a) is used, as well as the Kalman filter to combine the prior distribution of the model coefficients with the sample information. The non-standard optimisation routine works as follows: given an initial set of hyperparameters and its related optimisation statistic, the procedure interpolates a surface onto the statistics, determines its minimum and obtains the vector of hyperparameters associated with that minimum. When the statistic associated with the vector has been obtained, through the Kalman filter, the process of interpolation and minimisation is repeated, until convergence is reached. In this specific case, 200 iterations have been carried out.

The logarithmic transformation of all the series is considered in the model, except the rate of interest, which is expressed in basis points, and the budget deficit which is expressed as a percentage of GDP. The sample period used begins in the first quarter of 1974 and ends in the final quarter of 1996 (11). The number of lags considered in the various models was four (12).

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(10) Also as an alternative estimation criterion the likelihood has been maximised. Nonetheless, the out-of-sample predictive capacity is somewhat inferior to the use of an objective forecasting function.

(11) The period used to calibrate the model runs from the first quarter of 1974 to the last quarter of 1993.

(12) The consideration of a larger number of lags would reduce the predictive capacity of the model.

V

THE INTERRELATIONSHIPS BETWEEN THE MODEL VARIABLES

As mentioned in the first part of the paper, in VAR methodology analysis of the relationships between variables is usually based on study of the impulse response functions and the variance decompositions for the structural model specified.

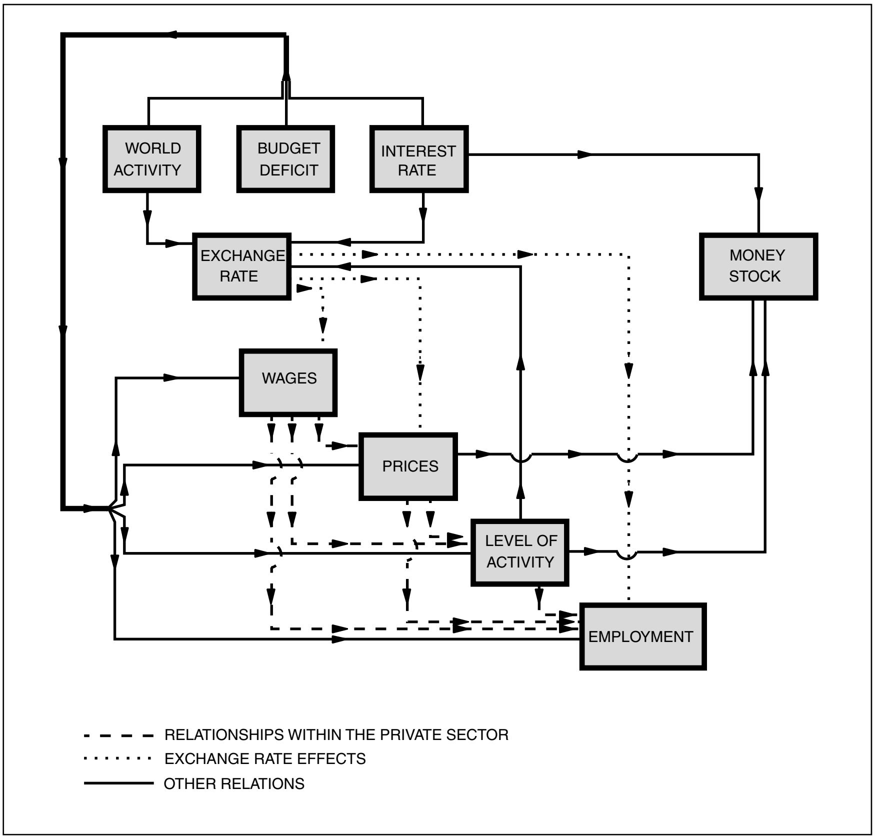
V.1. Reasons for the identification scheme used

There are different possibilities as regards the way in which identification of a model is achieved; basically these possibilities involve using contemporaneous, long-run or mixed restrictions (1). In this section contemporaneous restrictions are employed. These assume that certain variables do not affect others at the very moment the disturbance occurs, so that there is no contemporaneous causality in that direction. In any event, it should be emphasised that no restriction is placed on dynamic interrelationships between the different variables. Chart V.1 shows the economic restrictions specified, all defined contemporaneously. The direction of the arrow indicates the direction of the contemporaneous causality. So, for example, the arrow which goes from the interest rate to the exchange rate indicates that changes in the interest rate may affect the exchange rate contemporaneously. At the top of the chart are to be found the variables which are not affected contemporaneously by any other: world activity, the budget deficit and the interest rate. At a second level are to be found the exchange rate, affected by world activity, the interest rate and the level of activity, and the money stock, determined by the interest rate the price level and the level of activity. Finally, the private sector variables

(1) For the Spanish economy, Campillo (1992) and Campillo and Jimeno (1993) use Choleski schemes; Álvarez, Jareño and Sebastián (1993) and Ballabriga and Sebastián (1993) use non-recursive identification schemes based on contemporaneous restrictions; Álvarez and Sebastián (1998) use identification schemes based on long-run restrictions.

CHART V.1

REFERENCE IDENTIFICATION SCHEME



are at a third level, a recursive contemporaneous relationship being established between them in the order wages, prices, national activity and employment. In addition, the private sector variables are affected by world activity, the budget deficit, the interest rate and the exchange rate, although the latter does not affect the level of national activity contemporaneously.

This set of restrictions involves relinquishing an isolated identification of the structural disturbances of the economy in favour of an identification based on groups of disturbances (2). The identification by groups seeks to isolate sets of equations which may be taken as representative of the behaviour of specific economic agents. Thus, each set should capture in-

(2) See, as examples of this strategy, Ballabriga (1988) or Álvarez, Ballabriga and Jareño (1995).

dependent sources of variability (structural disturbances), so that the disturbances of the different groups must be orthogonal between each other. If the informative content of the data is considered not to be sufficient to isolate the behaviour represented by any of the equations of a group, the orthogonality may be obtained by establishing a recursive scheme between the variables of the group.

Under this approach, the identification scheme shown in Chart V.1 has five groups. The first two seek to isolate the disturbances associated with the two variables directly related to the external sector of the economy, namely world activity and the exchange rate. A third set would represent the behaviour of the public sector in its fiscal dimension. The fourth group would contain the behaviour of the money market, through consideration of the interest rate and money stock variables. Finally, the fifth set would include those variables which represent the output and employment aspects of the decisions of the private sector; this group would include the wages, prices, level of activity and employment variables. A more detailed description of these groups and their justification is given below.

The group formed by the world activity equation identifies the disturbance associated with this variable as structural. In other words, this identification means that the domestic variables of the Spanish economy cannot affect the level of world output contemporaneously (3).

The exchange rate is identified by means of an equation which opens channels for the contemporaneous effects associated with financial disturbances, through the interest rate, and trade disturbances, both domestic, caused by changes in the level of national activity, and external, associated with world activity.

The actions of the public sector are identified in the third group so that it is considered that the disturbances of the budget deficit are associated with fiscal policy. The identification used does not allow for the contemporaneous influence of any other variable on the budget deficit. So, for example, the effect of an increase in tax receipts as a consequence of an increase in activity shall be manifested with a lag. Moreover, fiscal policy is considered to act independently of the contemporaneous economic situation. This hypothesis is justified to the extent that the makers of fiscal policy obtain information on the current economic situation with a certain lag.

(3) Given the size of the Spanish economy, in the estimation of the different models the restriction has been imposed that the domestic variables cannot affect world activity, even with a lag.

TABLE V.1

ESTIMATED STRUCTURAL COEFICIENTS
BVAR Model (a)

$$\varepsilon_{GDP^*} = \nu_{GDP^*}$$

$$\varepsilon_E = \underset{(0.57)}{0.15}\varepsilon_{GDP^*} - \underset{(0.11)}{0.01}\varepsilon_I - \underset{(1.28)}{0.25}\varepsilon_{GDP} + \nu_E$$

$$\varepsilon_M = \underset{(0.03)}{-0.01}\varepsilon_I + \underset{(0.09)}{0.10}\varepsilon_{CPI} + \underset{(0.26)}{0.15}\varepsilon_{GDP} + \nu_M$$

$$\varepsilon_I = \nu_I$$

$$\varepsilon_D = \nu_D$$

$$\varepsilon_W = \underset{(0.09)}{0.04}\varepsilon_{GDP^*} - \underset{(0.02)}{0.02}\varepsilon_E + \underset{(0.02)}{0.02}\varepsilon_I + \underset{(0.10)}{0.08}\varepsilon_{PF} + \nu_W$$

$$\varepsilon_{CPI} = \underset{(0.15)}{0.22}\varepsilon_{GDP^*} - \underset{(0.03)}{0.13}\varepsilon_E - \underset{(0.03)}{0.03}\varepsilon_I - \underset{(0.18)}{0{,}11}\varepsilon_{NCF} + \underset{(0.18)}{0.20}\varepsilon_W + \nu_{CPI}$$

$$\varepsilon_{GDP} = \underset{(0.05)}{0.02}\varepsilon_{GDP^*} - \underset{(0.01)}{0.01}\varepsilon_I + \underset{(0.06)}{0.04}\varepsilon_D - \underset{(0.07)}{0.20}\varepsilon_W - \underset{(0.04)}{0.01}\varepsilon_{CPI} + \nu_{GDP}$$

$$\varepsilon_L = \underset{(0.15)}{-0.05}\varepsilon_{GDP^*} + \underset{(0.03)}{0.01}\varepsilon_E + \underset{(0.03)}{0.02}\varepsilon_I - \underset{(0.17)}{0.25}\varepsilon_D + \underset{(0.19)}{0.11}\varepsilon_W + \underset{(0.11)}{0.08}\varepsilon_{CPI} + \underset{(0.30)}{0.41}\varepsilon_{GDP} + \nu_L$$

Source: Authors' calculations.
(a) Overidentification test: : $\chi^2_9 = 7{,}10$, signification level 0.63. Standard deviation between parenthesis. ε: innovations of reduced form. ν: structural disturbances.

The fourth group of equations identifies the monetary sector, permitting separate analysis of disturbances arising from the supply and demand for money. Thus, the equation corresponding to the interest rate is associated with the money stock, which does not respond to the contemporaneous economic situation. This hypothesis, as in the case of fiscal policy, is a consequence of the lag with which information is received. The money demand equation is represented by the money stock equation, which, following a traditional view, depends on the level of activity, the price level and the interest rate.

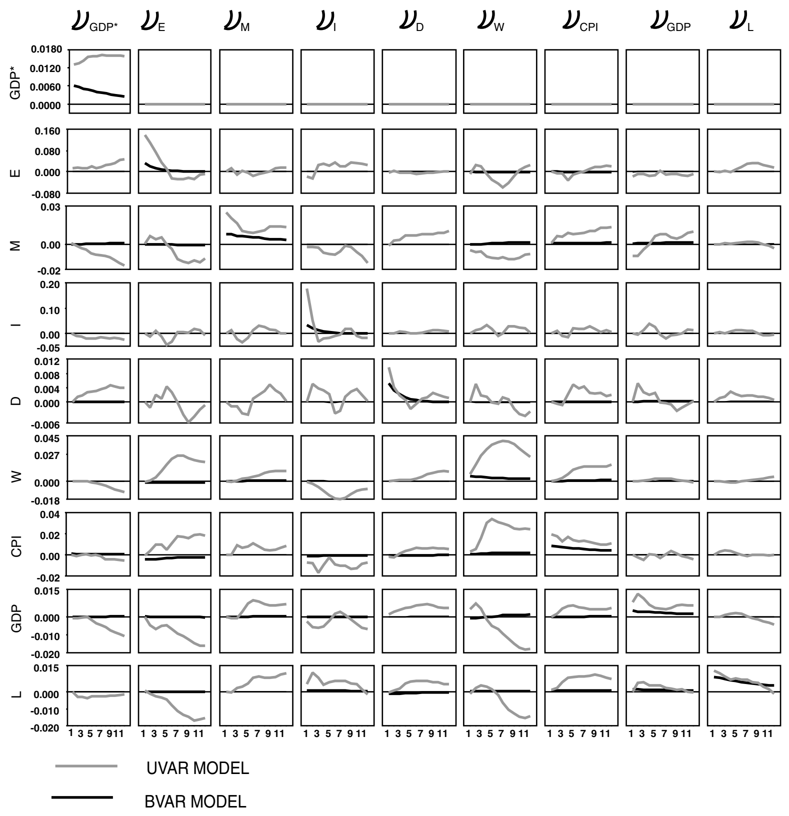
The private sector makes up the fifth group of equations of the identification. Given the set of variables included in this group, the product of the interaction between the supply and demand for goods, on one hand, and the supply and demand for labour on the other, the isolation of supply and demand disturbances for both markets seems excessively ambitious. It is in this context that the identification by groups of equations proves its usefulness, since, given the impossibility of isolating previous disturbances, the objective becomes to isolate the distrubances which affect the market for goods and labour as a whole. Thus, the disturbances associated with the equations for the level of activity, prices, employment and wages together represent the disturbances to the goods and labour market, without distinguishing between supply and demand, for goods and labour. The identification scheme specified permits this group of equations to react to fiscal, monetary and external disturbances. The possibility that some correlation persists between the disturbances of the block is eliminated by means of a recursive identification scheme, following the order of wages, prices, level of activity and employment.

In accordance with the reduced form estimated and with the identification scheme set out, the estimated contemporaneous structural coefficients and their associated statistics are shown in Chart V.1.

V.2. Transmission mechanism and contribution to variability

After the structural identification has been carried out, use of the impulse response functions and the variance decomposition permits the dynamic interactions of the model to be analysed. As discussed in the first part of this paper, the impulse response functions show the effects on the different variables of the system of identified disturbances, which could be interpreted as a *simulation* exercise indicating the sign, magnitude and persistence of the response. In turn, the variance decomposition indicates the contribution to the variability of the forecasting error for each variable, at different forecasting horizons, of each of the different distur-

IMPULSE RESPONSE FUNCTION
BVAR AND UVAR MODELS

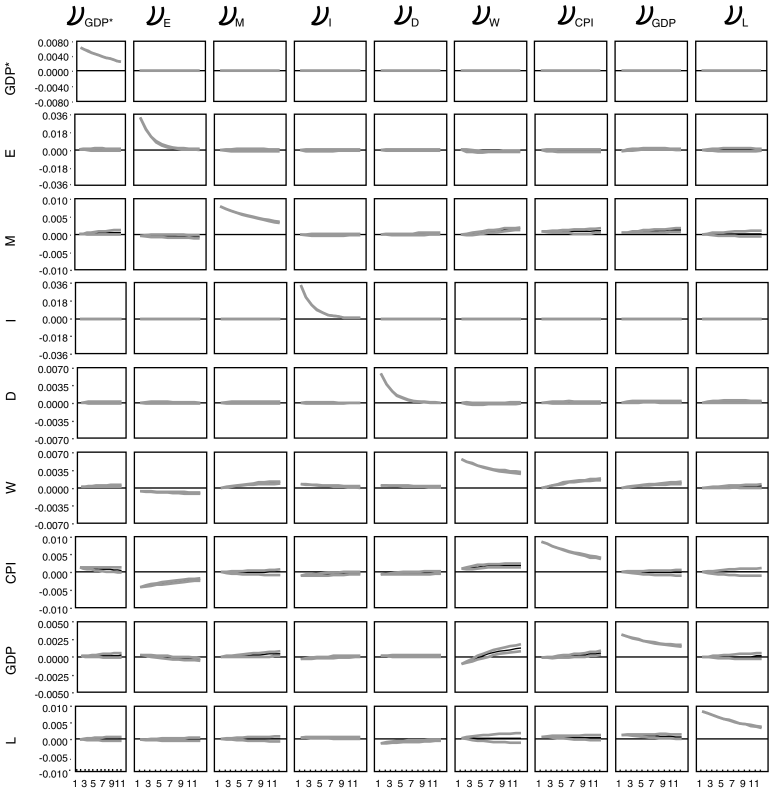


Source: Authors' calculations.
(a) ν_* represents the structural disturbance associated with the equation for *.

bances of the system. This section presents the results obtained for the forecasting model estimated in this paper (hereinafter, BVAR). The results for an unrestricted VAR model (hereinafter, UVAR) are presented by way of a counterpoint.

In the literature on VAR models it is customary to report in the impulse response functions the effects on each variable of movements in the structural disturbances with a magnitude of one standard deviation. Chart V.2 presents the impulse response functions obtained for the BVAR and UVAR models, which will enable a direct comparison to be made between the two models. Meanwhile, it is a widespread practice to

IMPULSE RESPONSE FUNCTION
BVAR MODEL



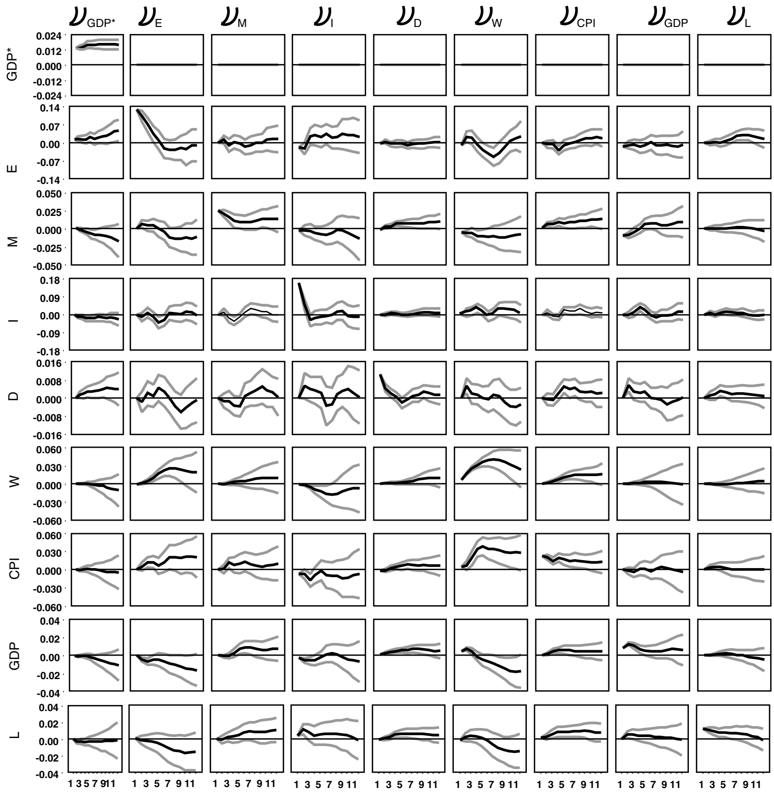
Source: Authors' calculations.
(a) ν_* represents the structural disturbance associated with the equation for *.

report impulse response functions accompanied by measurements of uncertainty, as discussed in part one. Charts V.3 and V.4 present these measurements for both models. In the light of the last three charts, the greater degree of interrelation in the UVAR model than in the BVAR model can be stressed. That said, a greater degree of interrelation is not necessarily desirable, since it could simply be reflecting a problem of overfitting; i.e. it may be the case that what is merely spurious interaction (noise) is being considered an interrelationship.

In the BVAR model impulse response functions the power and persistence of the effects of the own disturbances, as well as the lesser importance of cross effects can be appreciated. By contrast, the UVAR model

CHART V.4

IMPULSE RESPONSE FUNCTION
UVAR MODEL



Source: Authors' calculations.
(a) ν_* represents the structural disturbance associated with the equation for *.

impulse response functions reflect a greater degree of interrelation, as well as a high degree of variability in their dynamics. This difference in the degree of interrelation between models estimated by classical and Bayesian methods should come as no surprise, since it is customary in the literature. Nonetheless, to make claims regarding the goodness of the estimated interactions requires the use of some kind of criterion. The economic interpretation is one. The predictive capacity of the different models is another. As will be seen, both the economic interpretation and the predictive capacity of the BVAR model are preferable to those of the UVAR model. In this respect, it seems reasonable to maintain that the variability and the magnitude of the relationships of the UVAR model are

TABLE V.2

VARIANCE DECOMPOSITION
BVAR model (a)

Contribution of the structural disturbances to the variability of the forecasting error

	ν_{GDP^*}	ν_E	ν_M	ν_I	ν_D	ν_W	ν_{CPI}	ν_{GDP}	ν_L
GDP									
Short run	100.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
Long run	100.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
E									
Short run	0.2	98.7	0.1	0.1	0.1	0.4	0.1	0.1	0.1
	(0.2)	(0.6)	(0.1)	(0.1)	(0.1)	(0.4)	(0.1)	(0.1)	(0.2)
Long run	0.8	94.7	0.5	0.2	0.1	1.4	0.7	0.7	0.9
	(0.9)	(2.2)	(0.6)	(0.2)	(0.1)	(1.2)	(0.8)	(0.7)	(1.3)
M									
Short run	0.2	0.4	96.8	0.1	0.0	0.2	1.4	0.8	0.1
	(0.2)	(0.3)	(0.7)	(0.1)	(0.0)	(0.3)	(0.5)	(0.3)	(0.1)
Long run	1.1	1.1	87.6	0.2	0.2	3.1	2.8	2.8	1.1
	(1.2)	(1.0)	(3.4)	(0.2)	(0.3)	(1.7)	(2.2)	(1.7)	(1.4)
I									
Short run	0.0	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
Long run	0.0	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
D									
Short run	0.2	0.1	0.1	0.1	98.4	0.4	0.1	0.3	0.3
	(0.2)	(0.1)	(0.1)	(0.1)	(0.8)	(0.4)	(0.2)	(0.3)	(0.4)
Long run	0.9	0.3	0.7	0.2	93.4	1.0	0.6	1.3	1.6
	(1.1)	(0.3)	(0.8)	(0.2)	(3.0)	(0.8)	(0.8)	(1.3)	(1.9)
W									
Short run	0.3	2.6	0.3	1.4	0.6	93.9	0.7	0.2	0.0
	(0.1)	(0.3)	(0.1)	(0.1)	(0.1)	(0.3)	(0.1)	(0.1)	(0.0)
Long run	0.7	4.7	2.6	1.0	0.5	81.3	6.6	2.1	0.4
	(0.5)	(0.8)	(0.9)	(0.3)	(0.2)	(1.4)	(1.1)	(0.8)	(0.4)
CPI									
Short run	1.9	18.5	0.1	0.9	0.3	2.2	75.9	0.1	0.1
	(0.5)	(1.2)	(0.2)	(0.4)	(0.2)	(1.0)	(1.3)	(0.1)	(0.1)
Long run	2.2	17.8	0.5	0.9	0.3	5.9	70.7	0.6	1.1
	(1.4)	(2.4)	(0.9)	(0.6)	(0.3)	(2.4)	(3.5)	(0.9)	(1.4)
GDP									
Short run	0.2	0.3	0.1	0.6	0.3	5.9	0.1	92.4	0.1
	(0.2)	(0.2)	(0.1)	(0.2)	(0.2)	(1.1)	(0.1)	(1.3)	(0.2)
Long run	1.0	1.2	1.8	0.5	0.5	11.9	1.6	80.3	1.2
	(1.4)	(1.0)	(1.9)	(0.3)	(0.5)	(6.1)	(1.4)	(7.0)	(1.7)
L									
Short run	0.1	0.2	0.1	0.3	2.1	0.4	0.7	2.2	93.9
	(0.1)	(0.2)	(0.1)	(0.2)	(0.4)	(0.5)	(0.4)	(0.6)	(0.9)
Long run	0.6	0.5	0.8	0.5	2.0	2.9	1.4	2.9	88.5
	(0.7)	(0.6)	(1.1)	(0.5)	(0.9)	(3.4)	(1.4)	(2.1)	(4.2)

Source: Authors' calculations.

(a) Standard deviation in brackets. ν_* represents the structural disturbance associated with the equation for *.

VARIANCE DECOMPOSITION
UVAR model (a)

Contribution of the structural disturbances to the variability of the forecasting error

	ν_{GDP*}	ν_E	ν_M	ν_I	ν_D	ν_W	ν_{CPI}	ν_{GDP}	ν_L
GDP*									
Short run	100.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
Long run	100.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
E									
Short run	2.4	69.8	3.5	10.3	0.8	5.6	3.4	3.4	1.0
	(2.1)	(10.5)	(3.7)	(8.1)	(0.8)	(4.9)	(2.7)	(2.6)	(0.9)
Long run	9.1	34.3	7.5	17.2	1.7	13.4	5.1	7.0	4.9
	(8.3)	(11.2)	(5.6)	(9.0)	(1.1)	(7.8)	(3.9)	(4.3)	(3.3)
M									
Short run	2.5	7.1	52.5	8.3	3.0	9.4	7.0	8.8	1.4
	(2.7)	(5.7)	(11.5)	(6.2)	(2.2)	(6.7)	(5.5)	(3.4)	(1.8)
Long run	9.9	15.7	20.3	14.1	4.5	14.5	8.4	9.5	3.1
	(9.9)	(11.1)	(11.8)	(11.1)	(3.6)	(10.9)	(7.0)	(6.9)	(3.1)
I									
Short run	1.9	4.3	6.0	73.2	0.7	5.4	2.0	5.4	1.1
	(1.4)	(3.1)	(3.9)	(7.6)	(0.6)	(4.1)	(1.6)	(3.3)	(1.1)
Long run	5.0	12.4	9.6	43.2	2.0	11.2	5.2	8.6	2.9
	(4.4)	(6.7)	(5.6)	(9.6)	(1.1)	(5.7)	(2.7)	(4.3)	(2.0)
D									
Short run	4.3	11.1	8.7	20.8	22.6	11.7	4.5	12.0	4.3
	(4.2)	(9.4)	(7.3)	(13.3)	(7.4)	(7.8)	(3.5)	(7.4)	(4.2)
Long run	10.3	16.1	11.2	20.0	7.4	13.1	7.6	10.2	4.0
	(9.3)	(9.5)	(6.6)	(10.2)	(3.4)	(6.5)	(5.6)	(5.0)	(2.7)
W									
Short run	0.3	6.3	1.5	6.6	0.4	79.9	3.6	0.9	0.4
	(0.5)	(5.8)	(2.2)	(6.1)	(0.5)	(10.3)	(3.3)	(1.1)	(0.5)
Long run	6.5	17.9	6.8	13.5	2.7	35.2	6.8	7.5	3.1
	(8.8)	(14.7)	(7.7)	(11.5)	(2.5)	(17.6)	(5.3)	(7.9)	(4.0)
CPI									
Short run	1.1	8.7	7.5	15.4	1.2	31.1	28.4	4.1	2.5
	(1.3)	(7.8)	(7.2)	(9.8)	(1.0)	(16.5)	(9.9)	(3.6)	(2.9)
Long run	7.7	15.0	8.6	15.5	2.7	28.0	10.3	8.5	3.6
	(8.4)	(11.5)	(8.0)	(11.1)	(2.2)	(17.1)	(6.7)	(6.7)	(3.5)
GDP									
Short run	0.9	13.5	3.5	15.7	5.3	12.9	7.6	39.2	1.4
	(1.0)	(10.1)	(3.5)	(10.9)	(3.6)	(5.7)	(6.6)	(10.7)	(2.4)
Long run	8.2	19.9	10.8	12.7	4.9	22.4	5.3	12.5	3.4
	(10.6)	(14.2)	(8.8)	(10.6)	(3.9)	(15.9)	(5.9)	(8.2)	(4.4)
L									
Short run	3.4	8.7	6.8	24.2	3.1	8.7	10.1	9.7	25.4
	(2.9)	(7.4)	(7.4)	(14.9)	(3.0)	(8.8)	(8.6)	(8.3)	(9.3)
Long run	7.3	18.3	11.6	15.5	4.3	17.8	8.4	8.9	7.9
	(8.4)	(12.9)	(10.1)	(11.0)	(3.4)	(12.1)	(7.2)	(7.9)	(4.6)

Source: Authors' calculations.

(a) Standard deviation in brackets. ν_* represents the structural disturbance associated with the equation for *.

excessively affected by the non-systematic component of the sample information considered, and therefore reflect the existence of merely spurious effects.

With respect to their economic interpretation, the effects of the BVAR model are shown to be superior to those of the UVAR model. Thus, an increase in world activity has, in the BVAR model, an expansionary effect on the Spanish economy, which results in an increase in prices and the level of activity. By contrast, that same disturbance in the UVAR model would have a contractionary effect, which is not simple to explain. Likewise, exchange rate appreciations in the case of the BVAR model would have a deflationary effect, while, surprisingly, there would be increases in inflation in the case of the UVAR model. As for contractionary monetary policy actions, reflected in increases in interest rates, a decline in prices and in the level of activity is observed in both cases. Finally, a contractionary fiscal policy in the BVAR model reduces prices, while in the case of the UVAR model an increase in prices and in the level of activity is observed. As can be seen, the economic interpretation of the impulse response functions of the UVAR model is not at all satisfactory.

The second tool contemplated when analysing the dynamic interactions of VAR models is the decomposition of the forecasting error variance. Tables V.2 and V.3 present the results obtained for the BVAR and UVAR models, respectively; the short-term value indicates the variability explained at the end of the first year after the shock, and the long-term value corresponds to the end of the third year.

In the light of these tables, the conclusions drawn from the impulse response functions are corroborated; i.e. there is less interaction between variables in the BVAR model than in a UVAR model, although presumably, as already stated, for spurious reasons. Thus the BVAR model only displays effects of more than 10 % in the cases of the effects of the exchange rate on prices and of wages on output. By contrast, in the UVAR model effects exceeding 10 % are very numerous: those of world activity on the budget deficit; those of the exchange rate and the interest rate on all the domestic variables; those of the money stock on the budget deficit, output and employment; those of wages on the exchange rate and on all the domestic variables; those of prices on employment; and those of output on the budget deficit (many of them difficult to interpret on the basis of economic theory).

VI

SOME APPLICATIONS OF THE MODEL

This chapter presents various applications of the multivariate BVAR model estimated for the Spanish economy which attempt to exploit the information provided by the model, both for prediction and simulation.

VI.1. Forecasting performance

Once the quarterly macroeconometric model has been specified and estimated, it must be evaluated in terms of its forecasting performance. An initial evaluation can be based on the criterion used to estimate the model, i.e., since a one-year forecasting statistic has been used as the optimisation criterion, it should be checked that for the sample used in the calibration this model has the smallest error at that horizon. However, although the sample (1) for which results are given extends to the final quarter of 1996, while the data used in the calibration only extend to the end of the final quarter of 1993, the examination of the one-year forecasting statistic is still of interest.

Moreover, there are least three further aspects of interest: 1) how the model performs at horizons of other than one year; 2) how well the model forecasts particular variables, since the calibration statistic used is joint and global in nature. In this respect, since it is especially important for a central bank to obtain sound forecasts of inflation and economic growth, the forecasts obtained for these variables should be analysed in greater detail; and 3) the significance of the interrelationships between the different variables in the model. Although, generally, it is useful to evaluate the advantages of using multi-equation models instead of univariate models, in this particular case there are reasons to suspect that the advantages in terms of prediction are scant. In fact, as can be seen in Table V.1, the hy-

(1) Álvarez, Ballabriga and Jareño (1997) present additional evidence on the forecasting performance of the BVAR model for a different sample period.

perparameter which controls the uncertainty associated with the other variables has a low value (2). This, together with the low value of the hyperparameter associated with overall uncertainty (3), suggests that there is little interaction in the model and, therefore, that it is not very different from a model consisting of nine equations in which each variable depends solely on its past values.

To answer these questions three alternative models have been considered. The results of these models will be compared with those of the BVAR model. The first model considered is the unrestricted VAR model (UVAR model) used in section V.2. The second model, hereinafter referred to as BAR, as it contains Bayesian prior information and an AR specification, eliminates any interaction between the series, so that a set of equations is obtained in which each variable is determined solely by its lagged values. The third model considered is derived from using the prior distribution which reflects the behaviour regularities described in section I.3.3 (MIN). These three models constitute an appropriate reference framework for the BVAR model: when the BVAR model is compared with the UVAR model, the advantages of using a Beyesian approximation as opposed to a classical approach can be assessed; when the BVAR model is compared with the BAR model, the advantages of adopting a multivariate model as opposed to a univariate model can be weighed up; and finally, when the BVAR model is compared with the result of considering the prior information on behaviour regularities (MIN), it can be seen whether there are any advantages in carrying out a calibration process.

These models can be compared initially, as shown in Table VI.1, in terms of their goodness of fit, measured on the basis of the global forecasting statistics FE1, FE2 and FE3 (4).

Table VI.1 shows that the superiority of the BVAR model over the others is notable in all the statistics used. This shows that the overall forecasting performance of the BVAR model is better than that of the alternative models UVAR, BAR and MIN, not only at a horizon of one year, but for longer periods too.

That said, these statistics are global in nature, so that they do not discriminate between variables. However, as mentioned above, if one of the

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(2) Recall that the lower the value of this hyperparameter, the lower the importance in each equation of the past of the rest of the variables. In the limiting case where the hyperparameter is zero, each variable only depends on its own past and on determinant variables.

(3) Recall that this hyperparameter determines the relative weighting of the *prior* and sample information in the final estimation. The smaller the value, the greater the weight of the *prior* information.

(4) The definition of FE2 and FE3 is a simple extension for periods of two and three years of the statistic FE1 defined in section IV.2.

TABLE VI.1

OVERALL FIT OF THE MODEL (a)

Statistic	*Models*			
	BVAR	*UVAR*	*BAR*	*MIN*
FE1	**88.5**	118.2	115.42	108.3
FE2	**330.8**	527.8	430.50	508.7
FE3	**752.6**	1.340.6	955.63	1.610.2

Source: Authors' calculations.

(a) FE: mean square forecasting error statistic. The number refers to the maximum number of years considered in the calculation. The higher the value of FE the poorer the forecasting perfomance of the model.

main purposes of this model is to attain good forecasts of inflation and the other private sector variables, an individual analysis of each variable is necessary. One way of carrying out such analysis is to observe the mean absolute forecasting error of the different variables and models, as shown in Table VI.2.

The most notable results are the following: 1) in general, the BVAR model shows the best forecasting performance; 2) the unrestricted UVAR model displays a low predictive capacity, which is relatively worse the longer the forecasting horizon; 3) the MIN model, derived from the use of the prior distribution of empirical regularities, tends to offer forecasts which have very little to do with reality and also deteriorate badly for the longer periods, and 4) the BAR model tends to give fairly satisfactory results. If the results for the price variable are examined, the distance between the forecasts of the BVAR model -the most accurate- and those of the other models is notable. This same pattern is repeated when the forecasts of compensation per employee are analysed. On the other hand, in the period analysed in this table, the relative advantages of the BVAR model for the variables GDP and employment are smaller, the performance of the BAR model being markedly better in the case of GDP.

Given the importance of the analysis of inflation and GDP, it is worth studying in greater detail the characteristics of the forecasts generated by the different models considered. From the standpoint of analysing the economic situation, both the modification of the forecasts as new information is included and the profile of acceleration or deceleration of the forecasts are key elements in characterising trends in inflation and economic growth. With the aim of shedding some light on these features, the observed values and the forecasts of the BVAR model are compared with the forecasts of the UVAR model (see Chart VI.1), of the BAR model (see

TABLE VI.2

MEAN ABSOLUTE FORECASTING ERROR (a)
Period 1990:I - 1996:IV

CPI					*GDP*				
Horizon	*BVAR*	*UVAR*	*BAR*	*MIN*	*Horizon*	*BVAR*	*UVAR*	*BAR*	*MIN*
1	**0.25**	0.80	0.48	0.66	1	0.19	0.20	**0.18**	0.21
4	**0.49**	3.89	1.77	2.87	4	1.45	1.50	1.33	**1.22**
8	**1.05**	7.69	4.24	11.21	8	4.07	4.33	**3.80**	4.20
12	**1.95**	13.57	7.25	29.92	12	7.47	8.38	**6.62**	8.64
Employment					*Wages*				
1	0.64	0.76	0.64	**0.48**	1	0.41	**0.16**	0.63	0.39
4	2.46	2.60	2.47	**2.05**	4	**1.78**	2.45	2.88	1.83
8	6.71	**6.08**	6.71	6.41	8	**3.99**	9.12	5.58	6.54
12	**11.60**	12.96	11.62	14.44	12	**6.15**	16.07	7.05	19.90
Money Stock					*Exchange Rate*				
1	0.62	0.77	0.92	**0.61**	1	**1.65**	3.09	1.91	2.15
4	**1.75**	1.98	2.77	2.11	4	**4.00**	6.51	5.33	8.79
8	**4.35**	5.43	6.27	5.22	8	**6.74**	8.21	10.12	24.92
12	**6.80**	9.65	10.37	11.12	12	**9.54**	17.61	15.57	64.91
Interest Rate					*Budget Deficit*				
1	**0.75**	2.86	**0.75**	1.89	1	**0.50**	0.68	0.53	0.51
4	**1.94**	3.73	**1.94**	8.22	4	**0.92**	1.23	0.97	1.41
8	**2.75**	6.71	**2.75**	27.79	8	**1.52**	1.83	1.53	2.60
12	**4.20**	11.11	**4.20**	73.89	12	2.03	4.55	**1.75**	5.51
World Activity									
1	0.29	**0.25**	0.29	0.31					
4	0.88	**0.85**	0.88	1.18					
8	1.75	**1.26**	1.75	2.38					
12	3.07	**1.97**	3.07	3.67					

Source: Authors' calculations.

(a) The figures in bold correspond to the lowest values of the statistic for each variable and forecasting horizon.

Chart VI.2) and of the MIN model (see Chart VI.3). In each case, in addition to the observed values, the forecasts are presented with eight consecutive forecasting origins, the first group of series corresponding to the fourth quarter of 1994, and the last, to the third quarter of 1996 (5). Likewise, Table VI.3 shows the related mean absolute errors.

Comparison of the forecasts of the BVAR and UVAR models reveals the striking instability of the forecasts of the UVAR model, which is especially severe in the case of inflation. The erraticness of these forecasts is a consequence of the fact that the model overfits, and therefore extrapolates on the basis of relationships between variables which have a significant non-systematic component. In contrast, the BVAR model stands out both for the stability of the forecasts, as new information is included, and for the closeness of its forecasts to the values actually observed, even for long forecasting horizons. Comparison of the BVAR and BAR models also, on balance, puts the latter in a negative light. Despite not suffering from problems of instability, the forecasts of the BAR model are, in the case of inflation, very different from the actual rates. Finally, an analysis of the forecasts of the BVAR and MIN models shows that the MIN inflation forecasts are not at all accurate, whereas its forecasts of GDP are of a considerably higher quality.

Finally, if the mean absolute error in the period Q1 1990-Q4 1996 is compared (see Table VI.2) with that of the period Q1 1995-Q4 1996V (see Table VI.3), the better forecasting performance of the BVAR model stands out both in the case of the CPI and GDP and at the different forecasting horizons.

In short, the following conclusions can be drawn from the results presented.

1) In forecasting terms the BVAR model tends to be superior to the other models considered. This superiority is observed both at different forecasting horizons and for different variables.

2) The forecasting superiority of the BVAR model with respect to the alternative models considered is especially clear when the inflation rate is forecasted, when the differences are very significant.

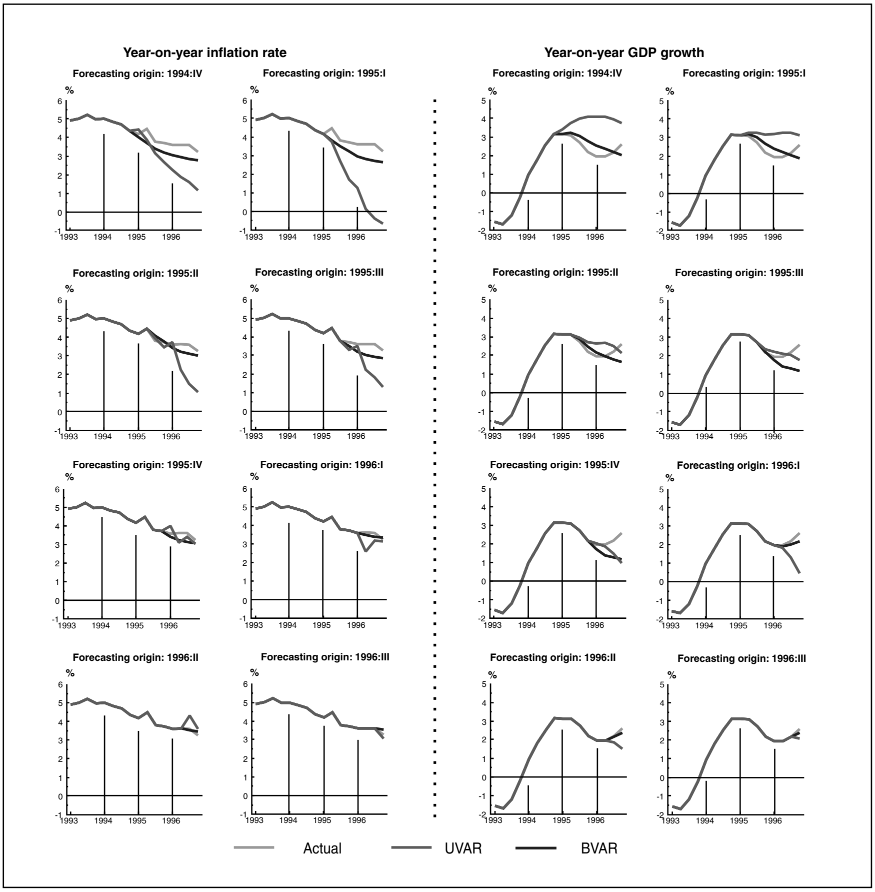
3) In general, there are advantages to be gained from using a multivariate rather than a univariate approach when forecasting. This suggests that the interrelationships captured by the model are important.

4) There are considerable advantages to be gained from an optimal selection of the prior information.

(5) The model is re-estimated with each observation. The prior distribution used does not vary.

CHART VI.1

COMPARISON OF BVAR AND UVAR MODEL FORECASTS (a)

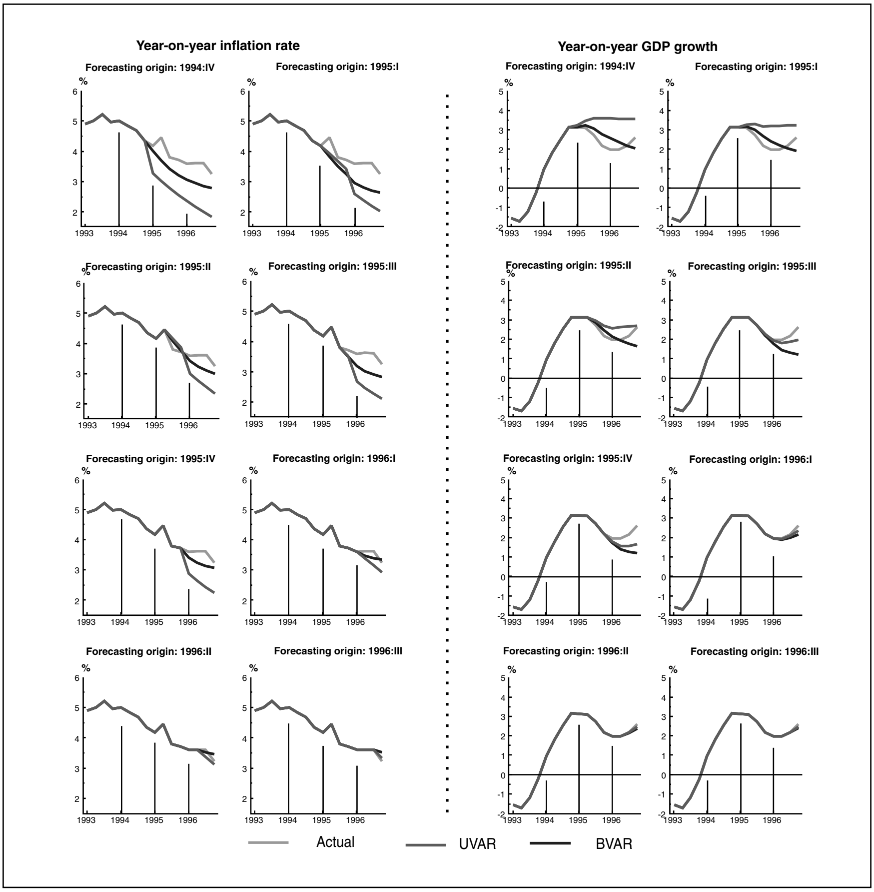


Sources: Instituto Nacional de Estadística and authors' calculations.
(a) The inflation data for 1995 are adjusted for the effect of changes in indirect taxation.

VI.2. The predictions of analysts and forecasts of the BVAR model

With regard to the applications of a forecasting model, attention is usually focused on the point forecast of the variables considered. In the previous section average forecasts were presented for the main macroeconomic magnitudes of the Spanish economy considered in this work, corresponding to different quantitative models. It is this type of forecast, moreover, which is usually published periodically by the various analysts who follow the Spanish economy. Accordingly, it is of interest to compare

COMPARISON OF BVAR AND BAR MODEL FORECATS (a)

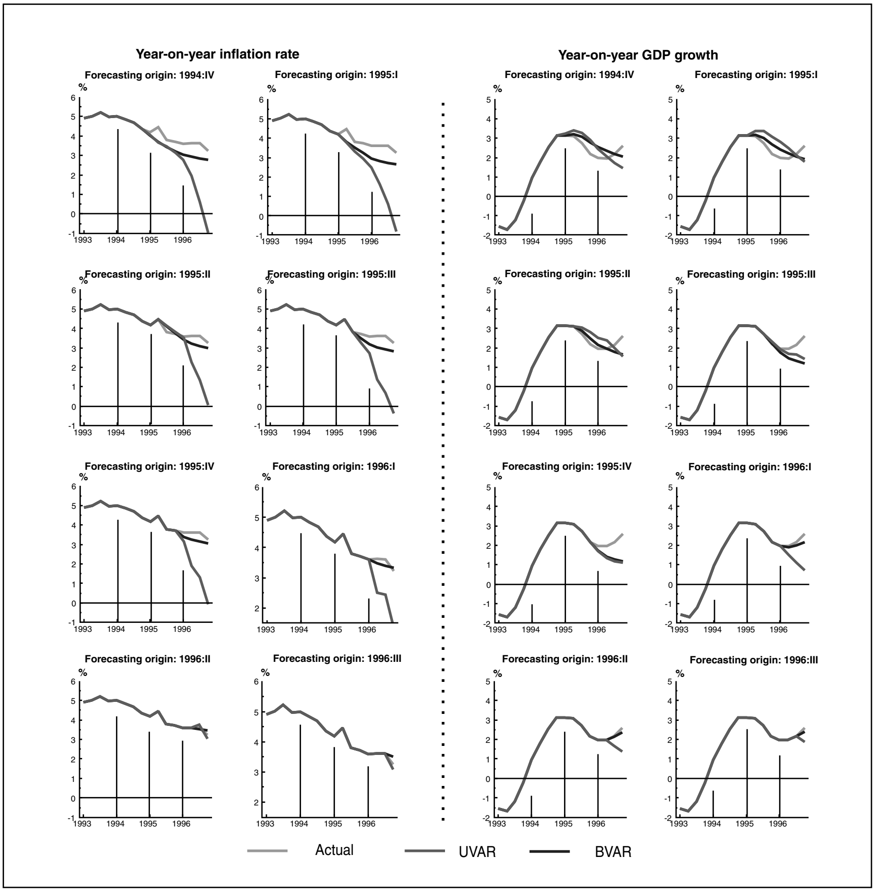


Sources: Instituto Nacional de Estadística and authors' calculations.
(a) The inflation data for 1995 are adjusted for the effect of changes in indirect taxation.

the results of the multivariate model BVAR with the forecasts made by different analysts. For this purpose, Chart VI.4 shows the errors in the inflation and growth forecasts made at end-1994 for the year 1995, both by the BVAR model and by a set of national and international analysts, and the exercise is repeated with the values forecast at end-1995 and those eventually observed in 1996 (6). If the set of forecasts made in both years is compared with the data eventually observed, the following points are

(6) The forecasts of the different analysts are taken from the January 1995 and January 1996 editions of the publication Consensus Forecasts.

COMPARISON OF BVAR AND MIN MODEL FORECASTS (a)



Sources: Instituto Nacional de Estadística and authors' calculations.
(a) The inflation data for 1995 are adjusted for the effect of changes in indirect taxation.

particularly striking: 1) all the analysts make errors in their forecasts of inflation and economic growth, which is consistent with the fact that forecasting is a complex task surrounded by uncertainty; 2) the general optimism of the forecasts for economic growth both for 1995 and 1996; 3) the pessimism of private analysts with regard to the possibility that the Spanish economy would see a notable reduction in the inflation rate in 1996; and 4) the forecasts of the BVAR models were closer to the values actually observed. It seems to follow, therefore, that the BVAR model provides different, and therefore useful, information from that published by other sources, using different tools.

TABLE VI.3

MEAN ABSOLUTE FORECASTING ERROR (a)
Period 1995:I - 1996:IV

CPI				
Horizon	*Models*			
	BVAR	*UVAR*	*BAR*	*MIN*
1 quarter	**0.24**	0.47	0.40	0.42
2 quarters	**0.32**	0.47	0.57	0.74
3 quarters	**0.35**	0.73	0.73	1.25
4 quarters	**0.47**	1.32	1.04	1.82
GDP				
Horizon	*Models*			
	BVAR	*UVAR*	*BAR*	*MIN*
1 quarter	**0.10**	0.22	0.12	0.33
2 quarters	**0.26**	0.56	0.29	0.63
3 quarters	**0.47**	0.97	0.57	0.85
4 quarters	**0.66**	1.10	0.88	0.79

Source: Authors' calculations.
(a) The figures in bold correspond to the lowest values of the statistic for each variable and forecasting horizon.

VI.3. Forecasting, uncertainty and the evaluation of targets

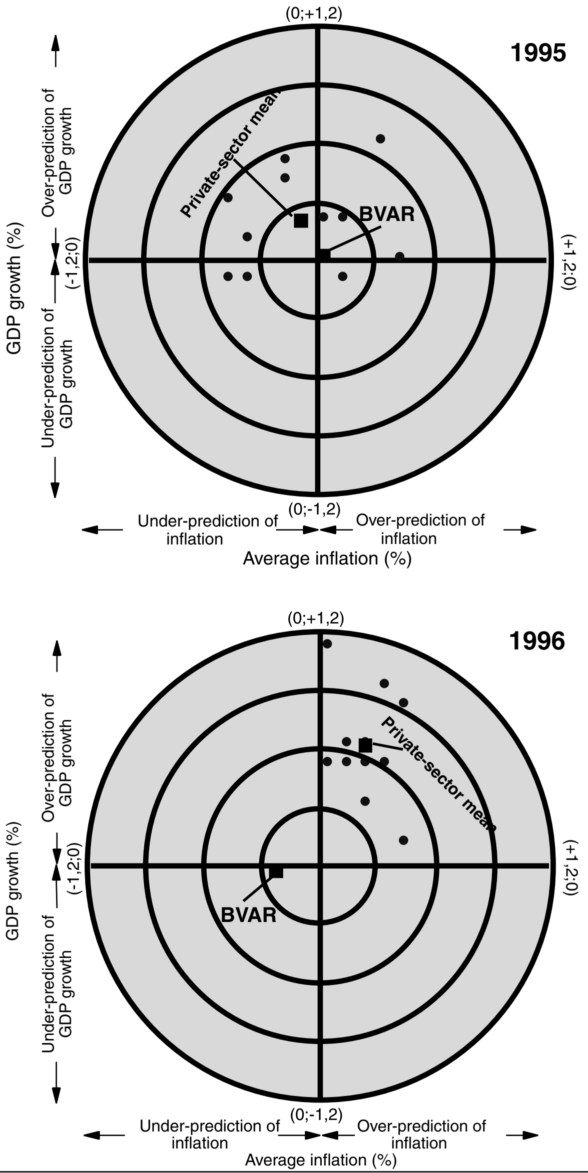
The forecasts used in Chart VI.4 were point forecasts (i.e. a single figure was given) of the future values of inflation and economic growth. However, as commented above, given non-negligible uncertainty which surrounds economic forecasts, no user should be satisfied with the presentation of a single figure for the future value of a macroeconomic magnitude and they should therefore demand a measure of the uncertainty associated with the mean forecast.

To have a measure of the uncertainty associated with the forecast is highly informative, because it allows not only the precision with which the forecast is made to be evaluated (the greater the uncertainty, the less relevant the point forecast), but also how different actual values are from the forecasts. In short, when attention is focused exclusively on point forecasts, very important information is disregarded that would help users to assess their accuracy and arrive at a fully informed opinion.

Despite these considerations, economic authorities, international organisations and private institutions rarely present forecasts accompanied

CHART VI.4

INFLATION AND GROWTH IN 1995 AND 1996 BVAR MODEL AND PRIVATE-SECTOR PREDICTIONS ONE-YEAR FORECASTING ERRORS (a) (b)



Sources: Consensus Forecasts and authors' calculations.

(a) Forecasting errors expressed as the percentage point difference from the actual annual average rate.

(b) The private-sector forecasts correspond to the January editions of the publication Consensus Forecasts. The institutions considered are:AB Asesores, AFI, Banco Bilbao Vizcaya, Banesto, BCH, CEPREDE, FG Valores y Bolsa, FIES, JP Morgan-Madrid and Universidad Carlos III.

by uncertainty measures. On occasions this may be due to a gap in the theory, since many types of models give no results for confidence intervals in terms of the growth rates of the series. This is the usual format in which macroeconomic forecasts, and in particular forecasts of GDP or the CPI, are presented. To illustrate the importance of giving a measure of the uncertainty associated with a forecast, Chart VI.5 presents, by way of example, the calculation of confidence intervals for the inflation and growth forecasts of the BVAR and UVAR models.

Specifically, the chart shows the values between which (according to the model, and with data to the fourth quarter of 1995) the projected inflation rate for 1996 and 1997 should lie with a probability of 25 percent (dark grey area), 50 percent (the light and dark grey areas), and 75 percent (all three areas of the chart). As can be seen, the uncertainty associated with the forecasts increases with the horizon of the forecast and is by no means negligible. Comparing the BVAR and UVAR forecasts, the high uncertainty associated with the projections of the latter model is notable, which considerably reduces the usefulness of its forecasts.

The usefulness of obtaining the probability distribution of the forecasts is not limited to providing a measure of their degree of reliability; it can also be used for other purposes such as assessing the probability that a variable will be above or below a given value. This application is especially important when a central bank sets direct inflation targets (7), as the Banco de España has been doing regularly since end-1994.

It is naturally of interest to assess the probability that the monetary objectives set will be attained. Once the probability distribution of the forecast inflation rate is known, the problem is one of estimating the cumulative probability within the target range. Chart VI.6 illustrates the nature of the problem. If an inflation rate below a given value is defined as a monetary policy objective (depicted in the chart as «target»), the probability that this objective is attained is expressed quantitatively by the value of the area below the probability density function for values less than the target value. Thus, this probability would be represented by the shaded area in the chart, being bounded from above by the probability density function and to the right by the target value.

For the purposes of illustration, Chart VI.7 shows how the probability of attaining the intermediate inflation reference fixed by the Banco de España (8) for the last few months of 1997 has changed. This reference in-

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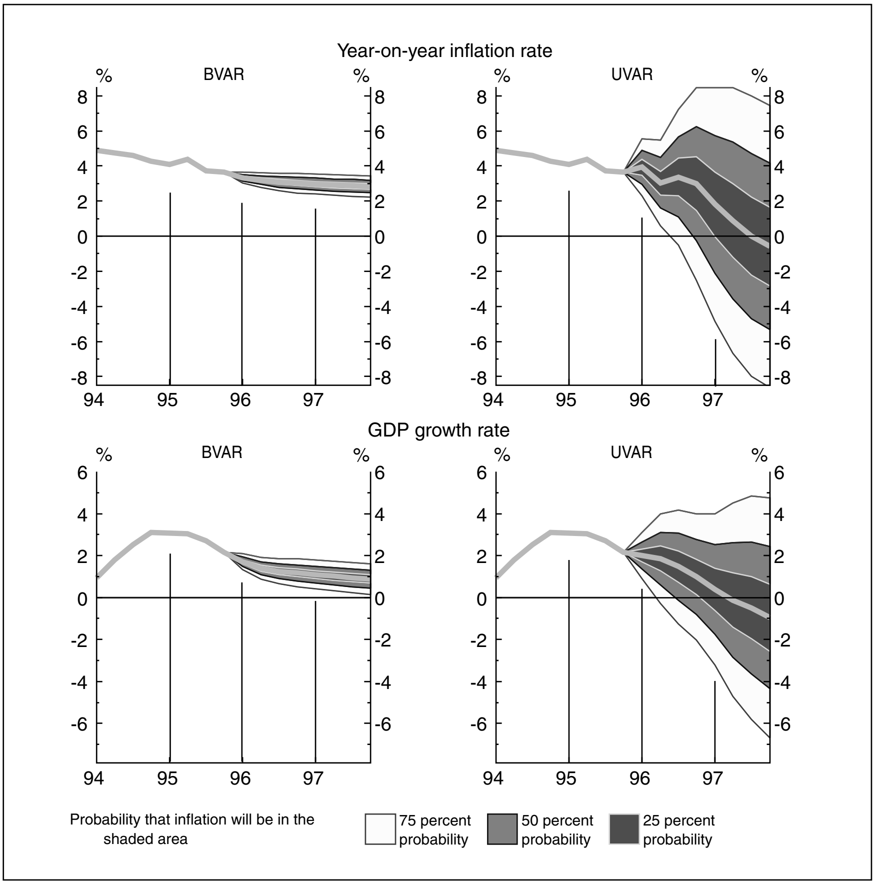
(7) Another no less important concern would be the establishment of a reference path for inflation that leads to attainment of the target set. An analytical method which can be used to estimate these paths is found in Álvarez, Delrieu and Jareño (1997).

(8) See Banco de España (1996).

CHART VI.5

UNCERTAINTY OF THE FORECASTS (a) (b)



Sources: Instituto Nacional de Estadística and authors' calculations.

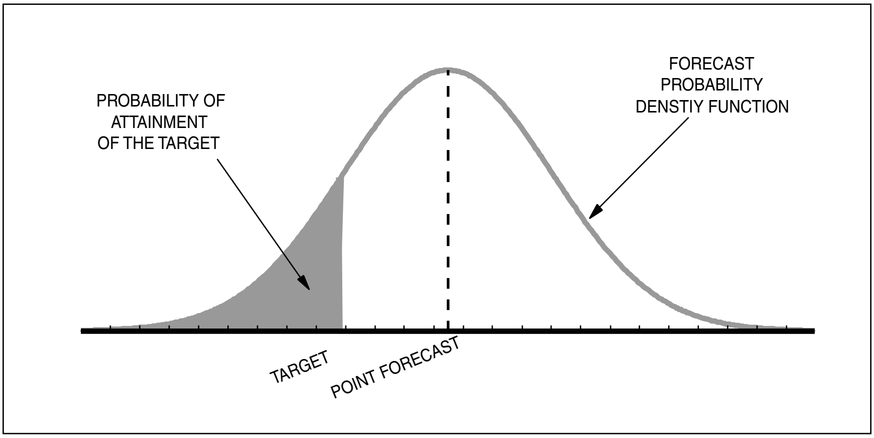
(a) Forecasts based on information to the fourth quarter of 1995.

(b) The shaded areas delineate the region of uncertainty for the forecast associated with their corresponding probability level.

volved placing the inflation rate at end-1997 close to 2.5%, so that the year-on-year growth of consumer prices during 1998 would draw close to 2%. Given that the reference was not established precisely, both a lax interpretation -placing the inflation rate in the final quarter of 1997 below 2.7%- and a strict interpretation -ensuring that the rate of change of consumer prices is below 2.5%- have been considered. It can be seen in Chart VI.7 that, on the information available in the third quarter of 1996 (when the target was still not defined), the probability of attaining it was around 23%, if a strict interpretation is adopted, and 36%, under a more lax interpretation, while in the fourth quarter of 1996 (with the target al-

CHART VI.6

PROBABILITY DISTRIBUTION AND ATTAINMENT OBJECTIVES

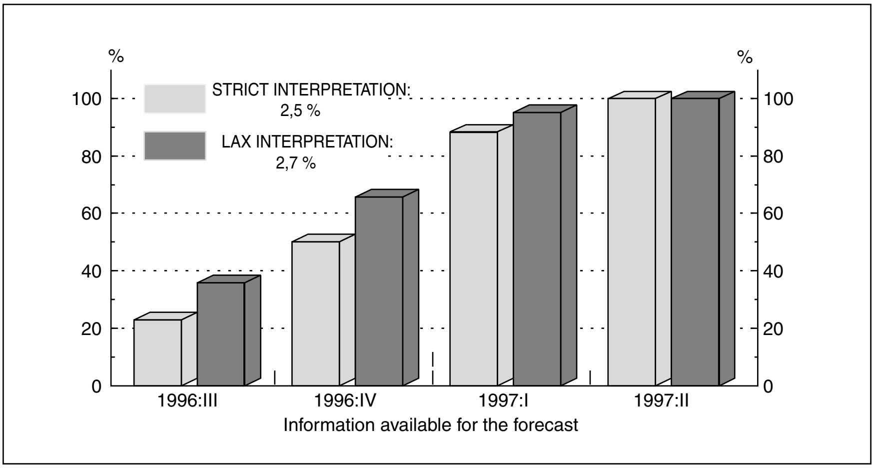


Source: Authors' calculations.

ready established), the probabilities of attainment were 50% and 66%, respectively, depending on whether a strict or lax definition is used. In the first half of 1997 the CPI trended very satisfactorily, the year-on-year rate falling from 3.2% in December to 1.6% in June. This sharp reduction led to a considerable improvement in prospects for inflation for the year as a

CHART VI.7

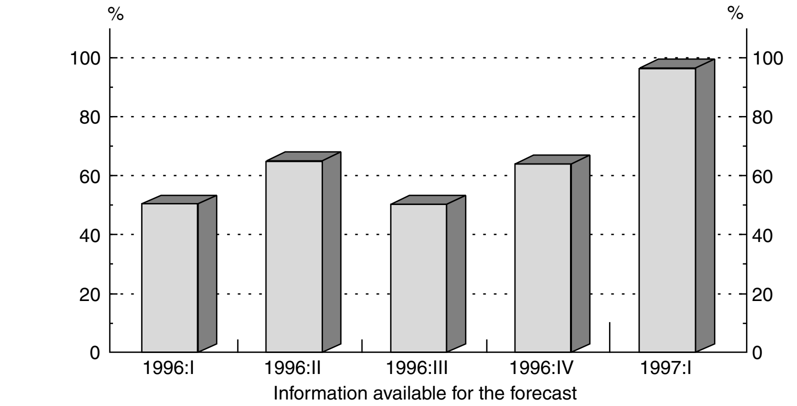
PROBABILITY OF ATTAINMENT OF THE INTERMEDIATE INFLATION REFERENCE FOR 1997



Source: Authors' calculations.

CHART VI.8

PROBABILITY OF ACCELERATION OF GROWTH IN 1997 (a)



Source: Authors' calculations.
(a) Acceleration of groth is defined as higher growth in 1997 then in 1996.

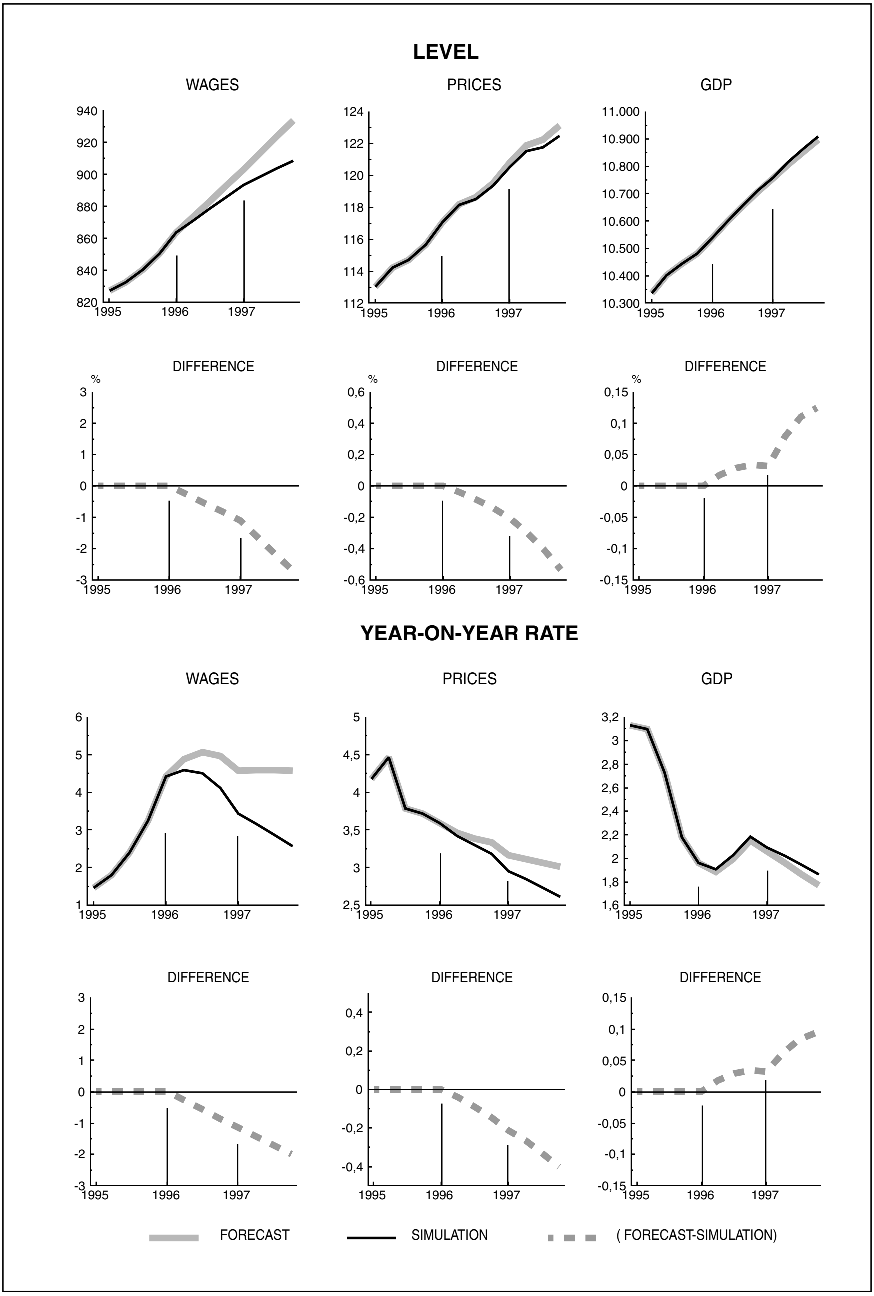
whole and a significant increase in the probability of meeting the objective. Thus, with a strict interpretation, in the first and second quarter of 1997 probabilities of 88% and 99.95% were reached, which rise to 95% and 99.9%, respectively, under a lax interpretation.

Besides enabling the probability of attaining the inflation targets set in the monetary programming to be assessed, there are numerous applications which derive from a full characterisation of the probability distribution of the forecasts. By way of example, Chart VI.8 shows the change in the probability that the growth forecast for 1997 would exceed that forecast for 1996, as the figures for 1996 and 1997 became available. From the chart it can be seen that the development of the Spanish economy during 1996 did not offer clear signs of acceleration in 1997. The data of the first quarter of 1997, however, involved a very notable advance in the growth process, clearly showing that the probability of acceleration of activity with respect to the previous year was close to 100%.

VI.4. Some simulations

One advantage of multivariate models like those considered above, over univariate models, is the possibility of performing simulations; i.e. estimating the effect on certain variables of changes in the paths of oth-

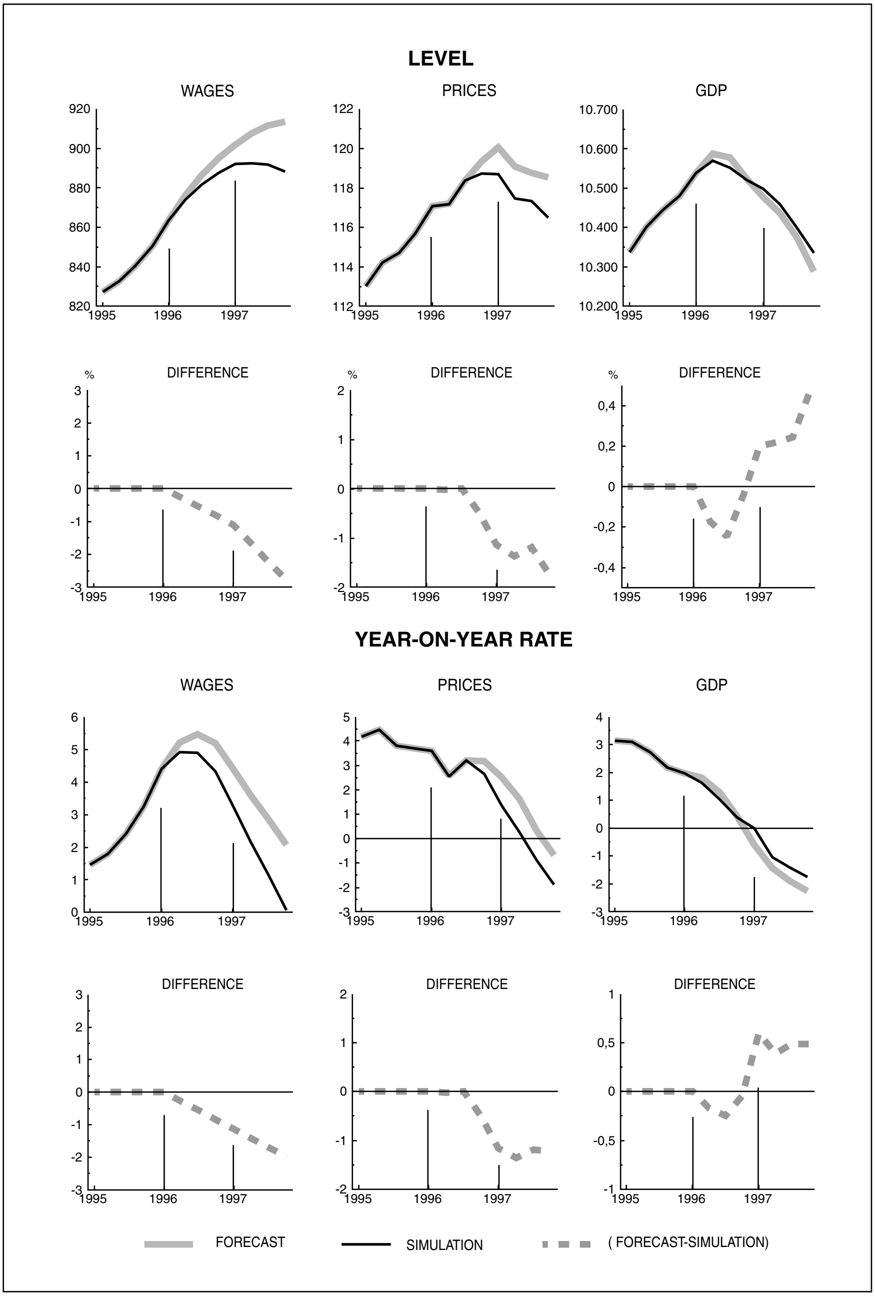
SIMULATION OF WAGE MODERATION
BVAR MODEL



Source: Authors' calculations.

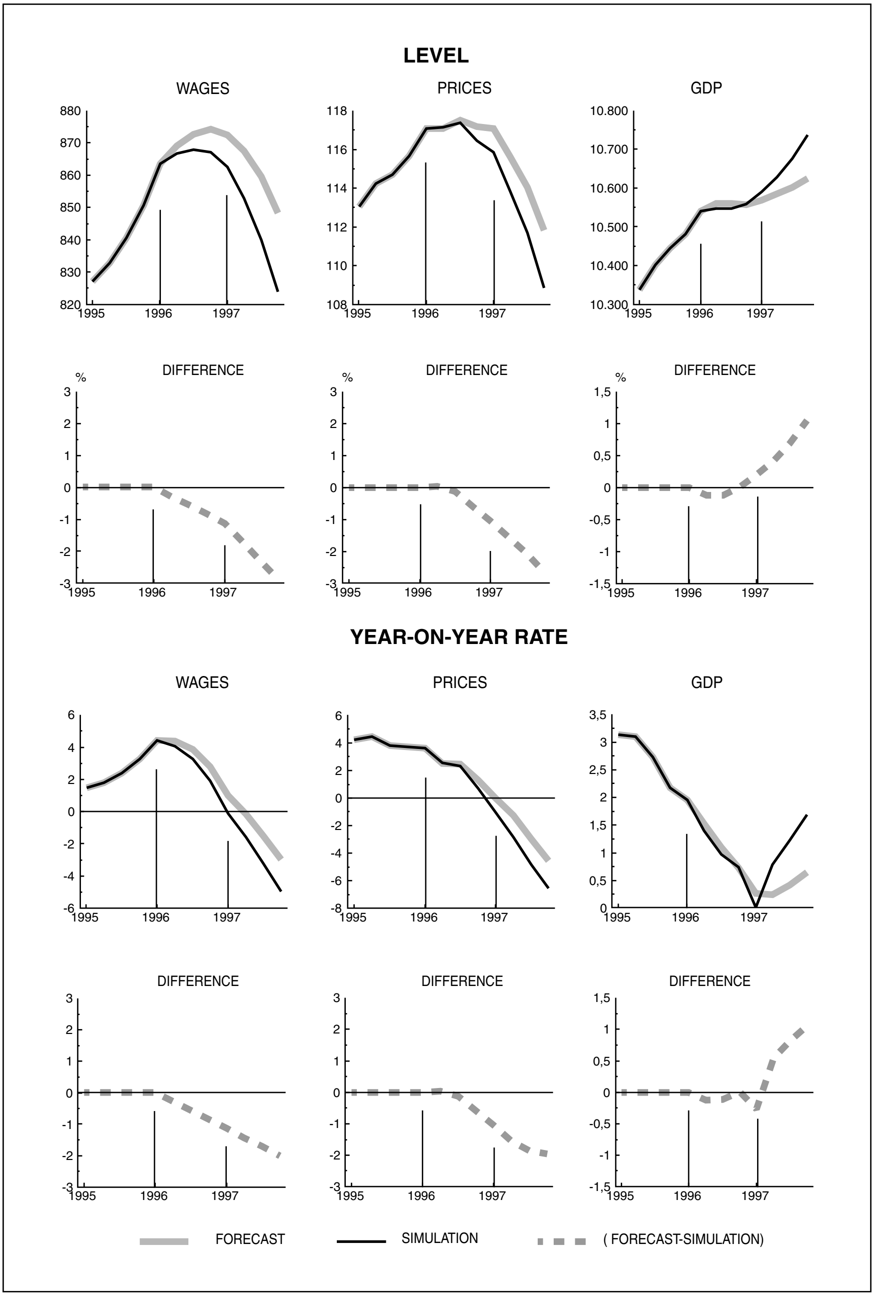
CHART VI.10

SIMULATION OF WAGE MODERATION
UVAR MODEL



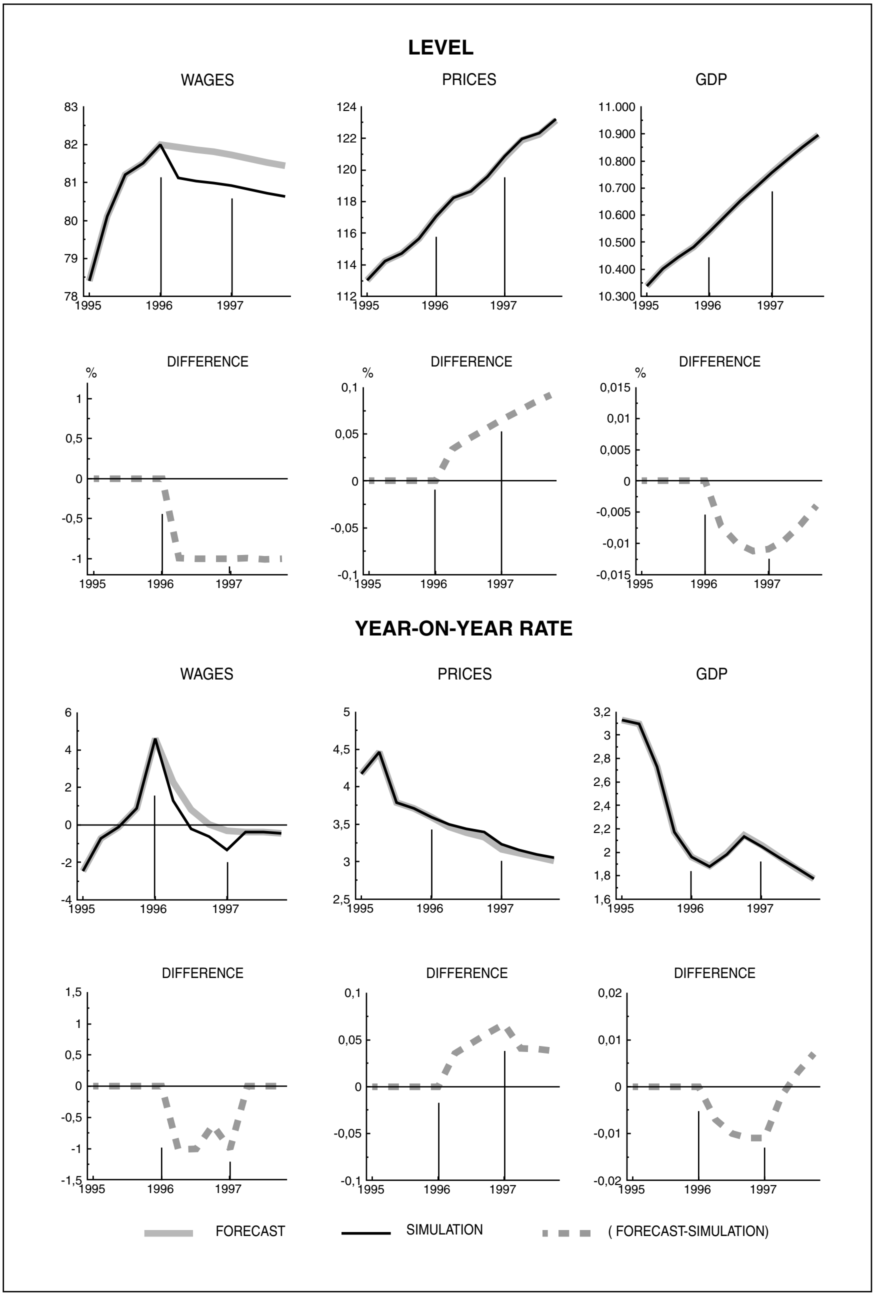
Source: Authors' calculations.

SIMULATION OF WAGE MODERATION
MIN MODEL



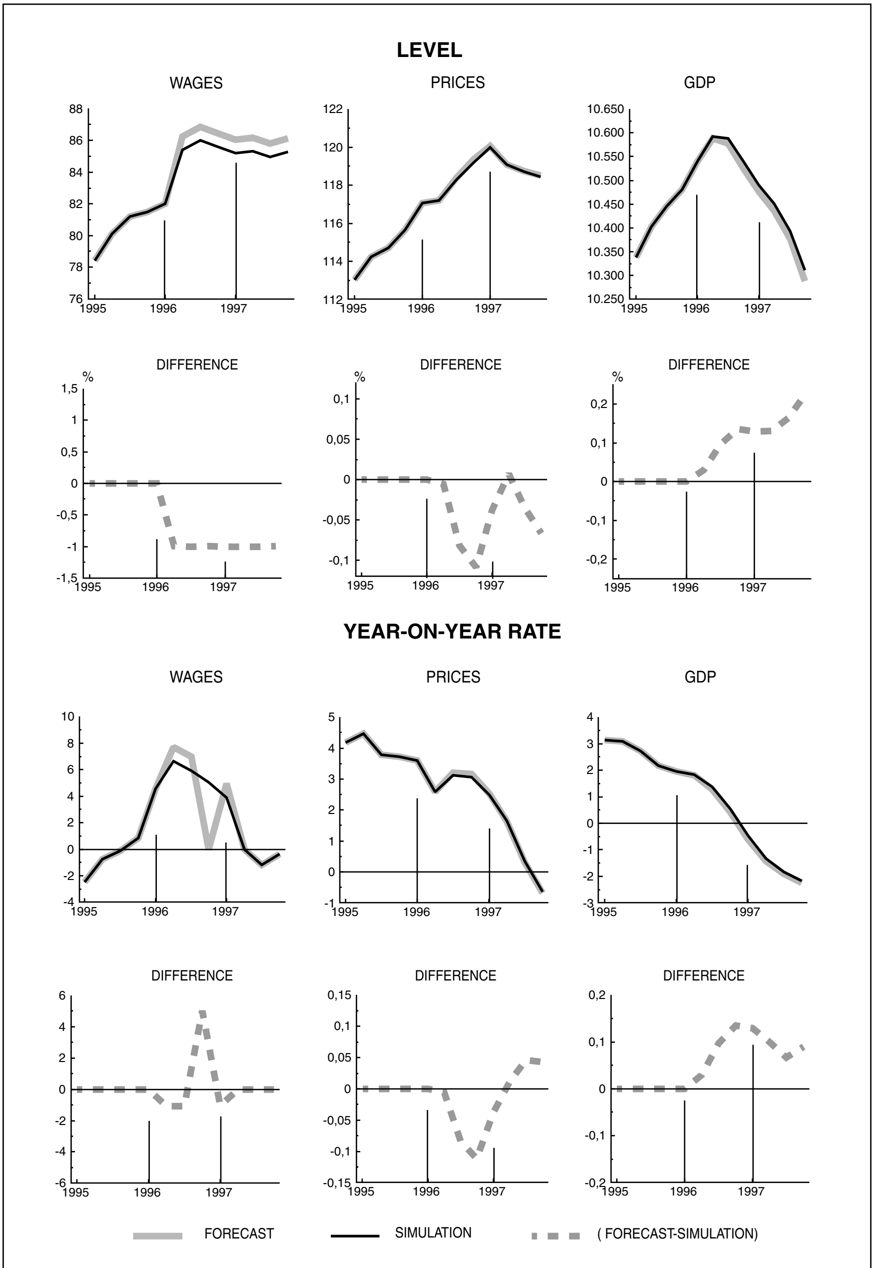
Source: Authors' calculations.

SIMULATION OF EXCHANGE RATE DEPRECIATION
BVAR MODEL



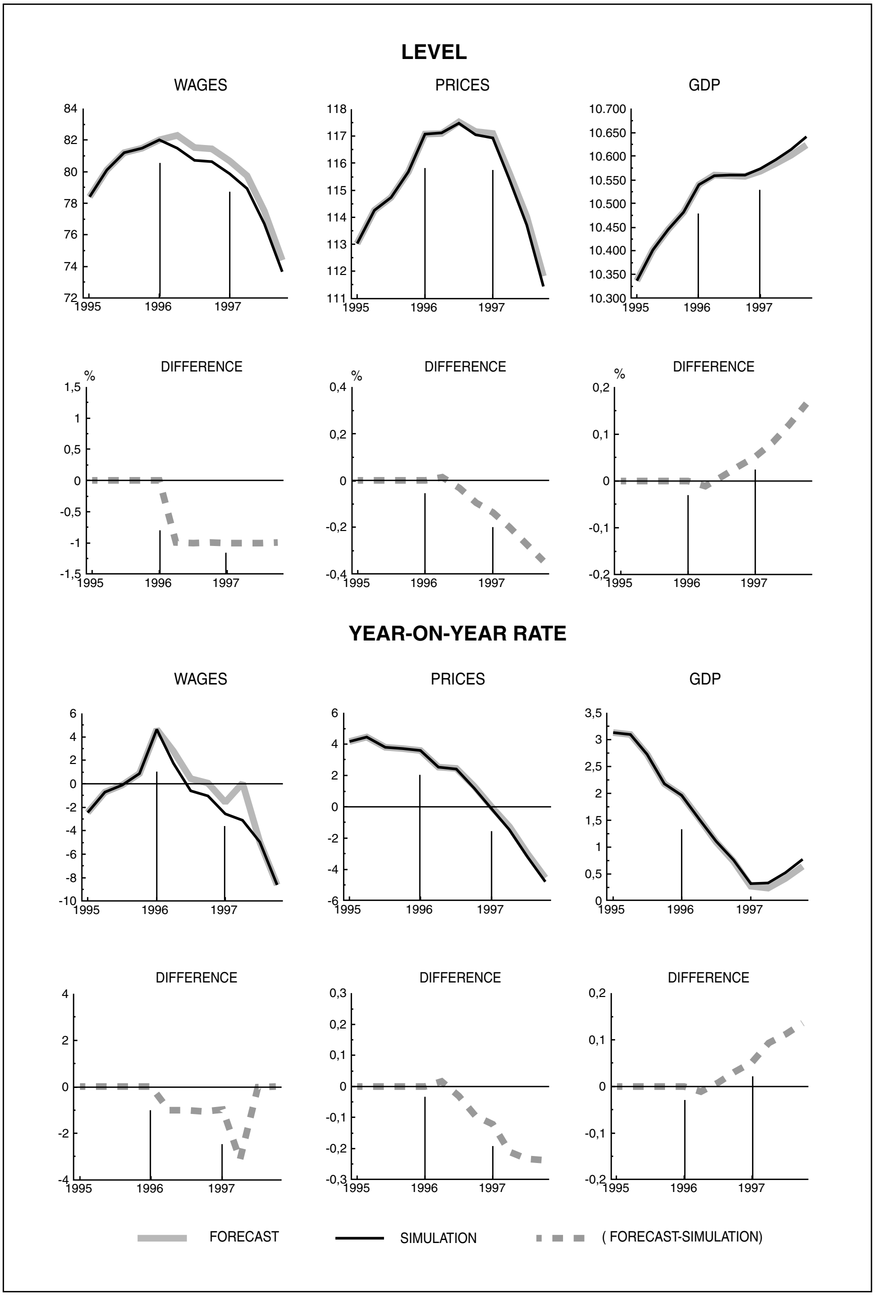
Source: Authors' calculations.

SIMULATION OF EXCHANGE RATE DEPRECIATION
UVAR MODEL



Source: Authors' calculations.

SIMULATION OF EXCHANGE RATE DEPRECIATION
MIN MODEL



Source: Authors' calculations.

ers. This section considers certain simulations using the BVAR, UVAR and MIN models, focusing, for the sake of brevity, on the effects on inflation and economic activity of changes in nominal wages and the exchange rate.

A frequently held opinion in macroeconomics is that nominal wage moderation is necessary in order to move towards a situation of price stability. To analyse the effect of a reduction in wage inflation in the different models considered a path is imposed whereby wage growth gradually slows, so that, after two years, its year-on-year rate stands two percentage points below the unconditional prediction. From a theoretical viewpoint, the reduction in nominal wages, in a context of some price rigidity, can be expected to produce a reduction in real wages thus making it profitable for firms to hire more workers and expand their output. It is this increase in supply which exerts a downward pressure on prices, so enabling the inflation rate to be reduced. Charts VI.9, VI.10 and VI.11 show the results of these simulations for the VAR, UVAR and MIN models. Besides the disparity in the unconditional predictions of the different models it is worth emphasising the following aspects:

1) the coincidence, in qualitative terms, of the conclusions of the various models;

2) the existence of an inflation-reducing effect;

3) the increase in economic activity, and

4) the disparity in the magnitude of the estimated effects.

The effect of exchange rate changes on prices and the level of activity is usually the object of considerable attention. From a theoretical viewpoint, although it is usually agreed that depreciations are generally accompanied by price rises, the response of the level of activity to a downward movement is ambiguous. On the one hand, a depreciation of the nominal exchange rate makes imported goods more expensive, at the same time as it cheapens domestic goods. The effect of the improvement in competitiveness is to increase net exports and output. On the other hand, a contraction in activity may be observed, as a consequence of the shift in aggregate supply. This is because a devaluation increases the price of imported intermediate goods used in productive processes. If there is no possibility of fully substituting these goods, there will be an increase in production costs which causes a contraction in supply. This, in turn, will tend to raise the prices of goods and reduce the level of activity.

To analyse the effects of a permanent devaluation of the nominal exchange rate, the effects estimated in the BVAR, UVAR and MIN models are shown in Charts VI.12, VI.13 and VI.14. The most notable features of the simulations are the following:

1) the qualitative disparity of the impact of the devaluation on prices, and

2) their coincidence in signalling an expansionary effect on the level of activity.

It might be particularly striking that both the UVAR and the MIN models find that a depreciation leads to reductions in the prices of domestic goods. However, it should be noted that the forecasting performance of these models is much less satisfactory than that of the BVAR model, so that the relationships captured between the different variables are much less reliable. Indeed, many authors maintain that little attention should be paid to the simulations obtained from models whose forecasting performance is unsatisfactory.

VII

CONCLUSIONS

The use of VAR methodology has grown in recent years, to become a tool commonly used by empirical macroeconomists. This expansion in its use has been based on its three basic features: objectivity, reproducibility and the systematisation of the way in which the econometric model is built. These features underlie the description of the methodology set out in part one of this book, which commenced with a brief historical outline, before going on to address the questions of formulation, specification, estimation and identification of VAR models. The description concluded with an explanation of its possible applications.

The second part of the paper described a BVAR macroeconometric model which has been used regularly by the Banco de España Research Department for projecting the main magnitudes of the Spanish economy, as well as to carry out simulation exercises, and which constitutes an instrument to assist in the making of economic policy decisions. The model was built to advance the empirical characterisation of the Spanish economy using Bayesian techniques and elements of time series analysis.

The exercises performed with the BVAR model and its comparison with alternative quantitative models produce a number of results. First, with a view to prediction, the use of Bayesian techniques is seen to be a useful way of extracting stable relationships between the set of macroeconomic magnitudes considered. Nonetheless, it is necessary to emphasise that simple Bayesian approaches (in which there is no calibration of the prior distribution, a distribution based on empirical rules for the behaviour of macroeconomic time series being used instead) fail to extract the most stable relationships. Meanwhile, it is confirmed that the interrelationships between the different variables considered help to forecast the future behaviour of the different series modelled -and in particular the inflation rate- with greater accuracy than if only their own past is considered; i.e. it is worth making the effort to move from univariate approximations to more complex multivariate approaches. Moreover, from a predictive viewpoint, it is worth pointing out that the forecasts derived from the

BVAR model for the Spanish economy are more accurate than the predictions made by a wide set of analysts.

Although the forecasting advantages of the model presented seem to be significant, another notable aspect of the BVAR model is its degree of interpretability. Both the dynamic relationships revealed by an examination of the impulse response functions and the results of the simulations presented seem to suggest that the interpretation of the BVAR model is, generally, more in line with the results of economic theory that that of the other models considered. Nonetheless, it should be stressed that, although the interrelationships detected allow more accurate forecasts to be obtained and the responses can be interpreted in the light of economic theory, such interrelationships are quantitatively small.

As has been emphasised in this paper, economic forecasting is an inherently difficult exercise, as reflected in the uncertainty normally associated with it. The logical response to this situation should be to try to characterise such uncertainty, instead of ignoring it and giving a false impression of rigour and precision. One advantage of closed econometric models, like the one used here, is that they allow the probability distribution of forecasts to be characterised. Moreover, once this distribution has been obtained, it is simple to evaluate the probability that certain events will occur, such as the attainment of inflation targets.

In short, the experience acquired from developing th∏e model and the results obtained appear to indicate that it is a useful tool for analysing and projecting the main macroeconomic magnitudes of the Spanish economy.

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