Dynamic Fiscal Competition with Residence Based Taxation*

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Abstract

Recent international agreements on tax data sharing aim to facilitate residence based taxation of capital and thus mitigate tax competition. I show that residence based capital tax rates can still decline with the number of financially integrated countries when public spending and debt are used strategically. While suboptimal in the steady state, strategic policies persist during transition if the scale of financial integration is low. Compared to coordinated policies, strategically set tax rates are lower while public spending and debt are higher in the short run. In the long run coordination implies lower capital tax rates and higher public spending.

JEL Codes: E61, H63, E62, F42, F62

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1 Introduction

Statutory capital tax rates have more than halved in OECD countries starting in the mid 80s.\footnote{The average OECD statutory corporate tax rate was above 48% in the mid 80s and reached 23.6% in 2011 (Source: OECD).} A large tax competition literature, building on the seminal work of Zodrow and Mieszkowski (1986) and Wilson (1986), has since emerged to rationalize this striking trend. From a theoretical perspective, an overwhelming focus on taxation in static models of balanced budget governments forecloses the interesting possibility that public finance in general, and taxation in particular, are in fact shaped by the strategic use of other policies. Indeed, the significant rise of public debt as a share of output in developed economies\footnote{During the period 1980-2007, the general government consolidated gross debt as a percentage of GDP has increased from 35.5% to 60.5% in the EU15, from 43.9% to 62.2% in the US and from 45.6% to 64.2% in Canada (Source: AMECO Database).} in the last decades suggests a wider menu of fiscal externalities, both static and dynamic, may be at work. Moreover, from a policy perspective, recent agreements on automatic tax data sharing between OECD/G20 countries\footnote{The Standard for Automatic Exchange of Financial Account Information in Tax Matters, released by OECD in July 2014 has been endorsed by 51 countries.}, together with other attempts to harmonize national taxation systems, are expected to counter tax avoidance and other forms of "harmful tax competition" that erode domestic tax bases. As these measures aim to restore the principle of residence based taxation in a world of integrated capital markets, it becomes important to understand how this taxation system interacts with the externalities induced by other instruments such as public spending and debt.

The paper sets out to make progress in this direction. I build a multi-country dynamic general equilibrium model and study the fiscal policy interactions arising between countries that share an integrated capital market but retain independence of their policies. Countries are assumed to be large, i.e., they take into account their effect on prices. Each country is inhabited by overlapping generations of two-period lived agents. Every period, national governments choose public spending and its financing through taxes and debt to maximize the welfare of the current generations. To focus on the role of deficit financed
public spending, I assume residence based capital taxation, so outright tax competition is
excluded. Nonetheless, capital mobility generates two distinct externalities. First, seeking
to attract mobile private capital, countries use productive public spending as an instru-
ment of fiscal competition.\footnote{Empirical research has shown that international capital flows respond not only to tax rate differentials but also to public spending outlays, such as infrastructure investments, research and development, or public services. See, among other, Devereux and Griffith (1998), Bénassy-Quéré et al. (2005), Redoano (2003), Bénassy-Quéré et al. (2007).} This increases the productivity of private capital and hence the output. Second, the option to finance this spending with debt, in addition to taxes, generates a negative pecuniary externality (an increase in the interest rate). This leads to higher public debt everywhere, which in turn crowds out private capital. The net welfare
effect of these fiscal interactions determines whether national governments implement co-
ordinated or strategic policies. Both externalities arise endogenously from the existence of
a common capital market which induces "beggar-thy-neighbor" policies and, qualitatively,
do not depend in any essential way on the finite horizon characterizing the government’s
problem.

In this framework I derive analytic policy rules describing strategic fiscal policies and
contrast them against policies implemented under international coordination, both in the
steady state and along a transition. I then analyze how fiscal externalities and equilibrium
policies depend on the scale of financial integration, proxied by the number of countries
participating in the common capital market. Finally, I compare the welfare properties of
strategic vs coordinated policies.

First, for a given number of financially integrated countries, fiscal competition leads in
the short run to lower capital tax rates relative to coordinated policies. A fundamental
result in standard tax competition obtains here in an essentially different environment. On
the one hand, taxation is residence based so there is no incentive to lower tax rates to
attract new capital. On the other hand, governments substitute current tax revenues with
debt in order to fund public spending that increases the tax base both by attracting more
capital and making it more productive. Therefore, lower tax rates under fiscal competition
do not translate necessarily into lower welfare in the short run, another major difference with respect to the standard tax competition framework. In contrast to the short run results, long run capital tax rates are higher and public spending is lower under strategic policies as capital accumulation is hampered by excessive public debt.

Second, I show that even in a world where residence based taxation is feasible, an increase in the number of financially integrated countries can lower capital tax rates. As the number of countries increases, fiscal competition becomes more intense, both public spending and debt go up. Since the latter increases faster, capital tax rates decline. In spite of lower taxation and higher productive public spending, the debt externality slows down capital accumulation both in the short run and in the steady state.

Finally, I find that relative to coordinated policies, fiscal competition is optimal in the short run from the point of view of the national government as long as the number of financially integrated economies is below some threshold. In this case, the positive effect of public spending dominates the crowding out effect of public debt. In contrast, coordination is welfare improving and therefore preferred by national governments in the steady state. Thus, the model predicts suboptimal policies may arise and persist in the short run if the scale of financial integration is limited.

The paper adds to the literature on policy coordination and fiscal competition. Within the former body of research, Chang (1990) studies public debt under capital mobility and concludes that the debt externality is increasing in the number of countries and that policy coordination is welfare improving. The analysis focuses on the steady state and abstracts from issues of capital accumulation. It also eschews fiscal competition, understood as bidding for mobile factors. In the current paper, these features give rise to new externalities and, as explained above, to crucial differences between transition and steady state equilibrium policies.

Kehoe (1989) shows policy coordination can be undesirable in the steady state when capital flight is possible since tax competition prevents confiscatory capital taxation. The
analysis excludes externalities from productive public spending and debt which are central to this paper. In particular, here fiscal competition may be preferred in the short run when the positive effects of the former outweigh the negative effects of the latter.

Moreover, the paper shares a concern for dynamics with some recent work on fiscal competition, such as Wildasin (2003), Rauscher (2005), Koethenbuerger and Lockwood (2010), Makris (2005), Batina (2009), Becker and Rauscher (2013), Gross (2014) or Klein and Makris (2014). Different from all these, the focus here is on residence rather than source based taxation and the simultaneous (and strategic) choice of public debt and productive public spending. Also, while many of these contributions analyze infinitely lived governments and steady state outcomes, the current paper looks at both short and long run policies set by myopic governments driven by political economy concerns. Moreover, the paper studies how policies and welfare change with the number of financially integrated countries.

The paper also contributes to the political economy literature on government debt. Complementing studies of closed economies (Cukierman and Meltzer (1989)), or small open economies (Song et al. (2012)), the framework presented here focuses on countries that are large enough to behave strategically. Thus, I study how cross border externalities act together with typical intergenerational conflicts in determining the equilibrium fiscal policy: young agents prefer low labor taxes, high capital taxes and high public spending that increases their productivity. Anticipating their old age political conflict over tax rates, current young attempt to spread the cost of public spending into the future, hence they favor public debt. Therefore, the better represented they are in the political process, the higher the number of countries and the indebtedness needed to justify coordinated policies.

The next section introduces the model. Section 3 defines and computes the equilibrium allocations under strategic and coordinated policies and section 4 compares these allocations both on a transition and at the steady state. Section 5 looks at the effects of an increase in the scale of financial liberalization. Section 6 calculates short and long run wel-
fare levels attained under the two policy regimes. The final section concludes. Derivations and proofs are relegated to appendices.

2 The model economy

Consider an infinite horizon economy that consists of \( n \) countries, indexed by \( i \), with identical technologies and initial conditions. Countries are populated by identical, immobile, two-period lived agents. In each country, population is stationary and normalized to one. Capital is perfectly mobile across the \( n \) countries.\(^5\) Each country has a government that taxes capital and labor and issues bonds to fund a productive public good. Competitive firms produce a unique, homogenous and costlessly tradable good whose price is normalized to one. This final good combines an endogenously determined variety of intermediate goods produced by monopolistically competitive firms using capital, labor and services stemming from the public good, provided at no cost by the government.

2.1 Households

When young, individuals supply labor inelastically, consume and save for the old age. An individual born at time \( t \) in country \( i \) maximizes the lifetime utility

\[
\max_{c^y_{i,t}, c^o_{i,t+1}} \ln c^y_{i,t} + \beta \ln c^o_{i,t+1}
\]

s.t. \( c^y_{i,t} = (w_{i,t} - s_{i,t})(1 - \tau^L_{i,t}) \),

\( c^o_{i,t+1} = s_{i,t} R_{t+1}(1 - \tau^K_{i,t+1}) \),

where \( c^y_{i,t}, c^o_{i,t+1} \) denote consumption flows, \( s_{i,t} \) are the savings of a young individual, \( w_{i,t} \) is the wage rate and \( \tau^L_{i,t} \) and \( \tau^K_{i,t+1} \) are the tax rates on labor and capital income, respectively.

Savings are tax deductible in the young age and the gross return \( R_{t+1} \) is taxed in

\(^5\)The mobile resource, generically called capital, can be thought to include human capital, knowing that skilled people are relatively more mobile, relaxing the assumption of immobile labor.
the old age. The tax deduction simplifies the analysis without loss of generality. Denote the marginal product of capital with \( q_{t+1} \) and the gross return on capital with \( R_{t+1} = 1 - \delta K + q_{t+1} \) where \( \delta K \) is the depreciation rate of capital. Assuming capital depreciates fully in one period, \( \delta K = 1 \) and \( R_{t+1} = q_{t+1} \).

Given policies, households’ optimal allocations are:

\[
\begin{align*}
    c^y_{i,t} &= \frac{1}{1 + \beta} w_i, t(1 - \tau^L_{i,t+1}), \\
    s_{i,t} &= \frac{\beta}{1 + \beta} w_i, t, \\
    c^o_{i,t+1} &= \frac{\beta}{1 + \beta} w_i, t(1 - \tau^K_{i,t+1}) R_{t+1}.
\end{align*}
\]

### 2.2 Production

Competitive firms in country \( i \) produce the final good using an endogenously determined range of intermediate goods \( x_{j,i,t} \) where \( j \in (0, v_i) \):

\[
Y_{i,t} = \left( \int_0^{v_i} x_{j,i,t}^{-\sigma} dj \right)^{1/(1-\sigma)}
\]

where \( \sigma \in [0, 1] \) is the inverse of the substitution elasticity between intermediate goods. Final good firms choose \( x_{j,i,t} \) given prices \( p_{j,i,t} \) to maximize profits \( \Pi_{i,t} = Y_{i,t} - \int_0^{v_i} p_{j,i,t} x_{j,i,t} dj \). This yields demand functions

\[
x_{j,i,t}(p_{j,i,t}) = p_{j,i,t}^{-1/\sigma} Y_{i,t}.
\]

The intermediate goods \( x_{j,i,t} \) are produced in a monopolistically competitive sector by firms that pay a fixed cost \( f \) every period to operate a technology that is constant returns to scale (CRS) in capital \( k_{j,i,t} \) and labor \( l_{j,i,t} \) and uses services stemming from a public
good, provided at no cost by the government:

\[ x_{j,i,t} = G_i^\delta k_{j,i,t}^\alpha l_{j,i,t}^{1-\alpha}, \quad 0 < \delta \leq \alpha < 1. \tag{7} \]

Each intermediate goods producer hires private inputs at given prices \( w_{i,t} \) and \( q_{i,t} \) to maximize profits

\[
\max_{l_{j,i,t}, k_{j,i,t}} \pi_{i,t} = p_{j,i,t} x_{j,i,t}(p_{j,i,t}) - (w_{i,t} l_{j,i,t} + q_{i,t} k_{j,i,t}) - f \text{ s.t. (6).} \tag{8}
\]

The first order conditions are:

\[
w_{i,t} = (1 - \sigma)(1 - \alpha) \frac{p_{j,i,t} x_{j,i,t}}{l_{j,t}}, \tag{9}
\]

\[
q_{i,t} = (1 - \sigma) \alpha \frac{p_{j,i,t} x_{j,i,t}}{k_{j,i,t}}. \tag{10}
\]

Substituting prices (9) and (10) back into the profit function (8) implies, together with the free entry condition, \( f = \sigma p_{j,i,t} x_{j,i,t} \).

In a symmetric equilibrium \( x_{j,i,t} = x_{i,t}, p_{j,i,t} = p_{i,t}, \forall j \in (0, v_{i,t}) \) and thus:

\[
Y_{i,t} = v_{i,t}^{1/(1-\sigma)} x_{i,t}, \tag{11}
\]

which, combined with (6), yields \( p_{i,t} = v_{i,t}^{\sigma/(1-\sigma)} \). Substituting the latter into the demand function (6) yields the equilibrium quantity of intermediate good \( x_{i,t} = f v_{i,t}^{\sigma/(1-\sigma)}/\sigma \).

Using this in (11) yields the expression for final output \( Y_{i,t} = f v_{i,t}/\sigma \). In a symmetric equilibrium \( k_{j,i,t} = K_{i,t}/v_{i,t} \) and \( l_{j,i,t} = N_{i,t}/v_{i,t} \) where \( K_{i,t} \) and \( N_{i,t} \) are the aggregate stock of capital and the population in country \( i \) respectively. Using these allocations in the production function for \( x_{i,t} \) (7) and solving for \( v_{i,t} \) yields the endogenous variety of intermediate goods:

\[
v_{i,t} = \left( \frac{\sigma G_i^\delta K_i^\alpha}{f} \right)^{(1-\sigma)/(1-2\sigma)}. \]
Thus, denoting \( z = (f/\sigma)^{1/2} \), \( \eta = \delta(1 - \sigma)/(1 - 2\sigma) \), \( \phi = \alpha(1 - \sigma)/(1 - 2\sigma) \) and using the normalization \( N_{i,t} = 1 \):

\[
Y_{i,t} = zC_{i,t}^{\eta}K_{i,t}^{\phi}.
\]

(12)

Factor incomes (9) and (10) are then given by:

\[
w_{i,t} = (1 - \sigma)(1 - \alpha)Y_{i,t} \quad \text{and} \quad q_{i,t} = (1 - \sigma)\alpha Y_{i,t}/K_{i,t}.
\]

(13)

Assumption 1. \( \eta + \phi < 1 \).

Assumption 1 implies overall decreasing returns to scale in reproducible inputs. Substituting the expressions for \( \eta \) and \( \phi \), it implies \( \sigma < (1 - \alpha - \delta)/(2 - \alpha - \delta) < 1/2 \). This also ensures that the number of intermediate goods increases with the stock of capital. Note that in equilibrium the aggregate output elasticity with respect to public spending is higher than the firm level counterpart (\( \eta > \delta \)). This is due to the indirect effect of the public spending on the entry in the intermediate goods sector and hence on the variety of such goods produced in equilibrium. This public spending externality, described more in detail below, opens the possibility of fiscal competition even with residence based capital taxation.

2.3 Government

In each country, the government uses three instruments to finance public spending: a tax on labor, a tax on capital and one period bonds, issued in the common capital market. Governments can commit to repay outstanding debt.\(^7\)

Importantly, residence based capital taxation is feasible. Thus, irrespective of where they invest their savings, the immobile households pay capital taxes only in the country of

\(^6\)See Chakraborty and Dabla-Norris (2011) for a more detailed discussion about the difference between the macro and the micro level output elasticity with respect to public spending.

\(^7\)Relaxing this assumption would imply that, in equilibrium, governments are able to borrow less. However, as long as debt remains positive, introducing symmetric commitment limits does not remove the cross border externalities that underlie the main results.
residence.

The budget constraint in period $t$ is

$$B_{i,t+1} + \tau_i^L w_{i,t} + \tau_i^K R_t s_{i,t-1} = G_{i,t} + R_{i,t} B_{i,t},$$

with $B_{i,0}, G_{i,0}$ and $s_{i,0}$ given, (14)

where $B_{i,t}$ is the outstanding debt at the beginning of period $t$. Solvency is ensured by the transversality condition $\lim_{T \to \infty} \left( \prod_{t=t_0}^T R_t \right)^{-1} B_{i,T} = 0$, $\forall t_0 > 0$.

Every period, fiscal policy allocations are chosen by a government maximizing the following social welfare function:

$$U_{i,t} = \chi u_{i,t}^y + (1 - \chi) u_{i,t}^o,$$

where $u_{i,t}^y = \ln c_{i,t}^y + \beta \ln c_{i,t+1}^o$ is the lifetime utility of the currently young agents and $u_{i,t}^o = \ln c_{i,t}^o$ is the utility of currently old agents. $\chi \in (0, 1)$ and $1 - \chi$ denote the weight of the young and old generation respectively. The old-age welfare of agents who are young in period $t$ enters the aggregate welfare function both in period $t$ and period $t + 1$, the first occurrence being due to the forward looking behavior of young agents.

It is straightforward to rationalize this social welfare function as the outcome of a probabilistic voting model. The mechanism is standard in the literature so I only describe it briefly.\(^8\) Probabilistic voting assumes the existence of a separate "ideology" dimension, orthogonal to the policy variables. With two political parties maximizing their expected vote share, the probability that a vote is cast in favor of one party is a continuous, increasing function of the relative appeal of that party’s platform. In general, as shown in Persson and Tabellini (2002), the equilibrium proposed policies are identical and maximize a weighted sum of agents’ welfare.

The focus of the paper are the fiscal externalities that arise from public spending and

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\(^8\)See Persson and Tabellini (2002) for a detailed exposition of probabilistic voting models. Applications to fiscal policy include Dixit and Londregan (1998), Strömberg (2004), Hassler et al. (2005), Gonzalez-Eiras and Niepelt (2008), Song et al. (2012).
debt when multiple economies are connected through a common capital market. As such, these externalities do not depend qualitatively on the finite time horizon that characterizes the government’s problem. Therefore, generalizing the model along this dimension would only make the analysis more involved without changing substantially the main insights.

2.4 The integrated capital market

The $n$ countries share a common capital market accessible to both firms and governments. Denote aggregate variables as $X_t = \sum_{i=1}^{n} X_{i,t}$, for $X = \{Y, K, G, c^y, c^o, s, B, \}$. Every period, the common capital market clearing condition reads:

$$K_t = S_{t-1} - B_t. \tag{16}$$

2.5 The fiscal externalities

The integrated capital market yields two distinct externalities arising through: (1) government debt and (2) public spending.

The public debt externality is straightforward. In a closed economy, higher public debt crowds out capital and increases the interest rate. Thus, the cost of debt is partially internalized, even by myopic governments. When borrowing in an international market, however, governments ignore the fact that an increase in the interest rate crowds out private capital and lowers output in all other countries. Thus, independently set fiscal policies rely too much on borrowing.

Importantly, the pecuniary debt externality occurs independently from the agents’ finite life-spans. Benevolent governments representing infinitely lived agents still issue excessive amounts of debt when part of a common capital market.

The second externality arises through public spending. While capital flows freely across countries, the owners are immobile. In order to focus on the interplay between public spending competition and debt, in this paper, I further assume the capital taxation is
residence based, so that the direct, intraperiod, tax competition channel is shut down.\(^9\)

Thus, capital mobility requires the marginal product of capital before tax to be equal across countries, i.e.:

\[
\frac{G_{i,t}^{\eta}}{K_{i,t}^{1-\phi}} = \frac{G_{j,t}^{\eta}}{K_{j,t}^{1-\phi}}, \forall i \neq j. \tag{17}
\]

Competitive capital markets and full depreciation further imply the return on assets is equal to the international marginal product of capital, \(R_t = q_t\).

The presence of a publicly provided input in the production implies that the marginal product of capital can be affected by fiscal policy and that governments choose \(G_{i,t}\) strategically to attract private capital, given the choices of other governments.

Thus, when the income of the old agents is taxed in the country of origin and the pre-tax returns are equalized, rewriting (17) for all countries yields the following equilibrium condition

\[
K_{i,t} = g_{i,t}K_t, \text{ where } g_{i,t} = G_{i,t}^{\frac{1}{1-\phi}} \left( \sum_{j=1}^{N} G_{j,t}^{\frac{1}{1-\phi}} \right)^{-1}. \tag{18}
\]

where \(K_t\) is the common market aggregate stock of capital.\(^10\) Intuitively, the stock of physical capital that each country can attract depends on its share in total public spending and the total capital stock available in the integrated economy. This relationship summarizes the fiscal competition among countries in each period. Using (18), the production function in each country can now be expressed only in terms of the public spending in all countries and the aggregate capital stock

\[
Y_{i,t} = zG_{i,t}^{\phi}(g_{i,t}K_t)^{\phi}. \tag{19}
\]

\(^9\)This is not very restrictive, since, as shown by Armenter and Ortega (2010), if residents or firms are mobile, the effects of tax competition on the tax revenues may offset each other - as the lower tax rate is compensated by a higher tax base. Razin and Sadka (2004) analyze the effects of tax competition in a political economy model. They allow the capital income to be taxed both in the home and the foreign country.

\(^{10}\)This condition is very similar to the payoff function postulated by Bucovetsky (2005) in a model of public input competition. Here, it arises naturally from the assumptions of strategic public investment and integrated capital markets.
Given fixed costs related to entry of $\sigma Y_t$, the aggregate resource constraint of the $n$-country economy is

$$(1 - \sigma)Y_t = C_t^p + C_t^o + K_{t+1} + G_t. \quad (20)$$

3 Policy regimes and equilibrium allocations

Let $\Theta_{i,t}(s_{i,t-1}, b_{it})$ denote the policy vector ($\tau_{i,t}^L, \tau_{i,t}^K, G_{i,t}, B_{i,t+1}$) as functions of the state variables in country $i$ at time $t$. Given public policies, $\Theta_{i,t}$ the private sector equilibrium is given by (2), (3), (4), (9), (10) and (17).

In the following I study two policy regimes, focusing on symmetric Markov perfect equilibria. Under the first regime, termed "strategic policies" governments choose national policies independently in order to maximize the utility of domestic agents given other countries' policies. Under the second, termed "coordinated policies", fiscal policies are set to maximize the welfare of currently living generations in all countries subject to all public budget constraints. Next I define equilibrium under each of these policy regimes.

3.1 Strategic policies

Denote strategic policies by superscript $s$. Each country $i$ solves

$$V_{i,t}^s = \max_{\Theta_{i,t}} U_{i,t}, \quad (21)$$

s.t. $B_{i,t+1} + \tau_{i,t}^L w_{i,t} + \tau_{i,t}^K R_t s_{i,t-1} = G_{i,t} + R_{i,t} B_{i,t}, \quad (22)$

given policies in other countries $\Theta_{j,t}, j \neq i$, households’ and firms' optimal decisions, (2), (3), (4), (9), (10), and states $s_{i,t-1}, B_{it} i \in \{1, 2, \ldots n\}$ (which, through (16) define $K_t$), and the common interest rate given by (17). Note that welfare in country $i$ depends on policies in the rest of the world through the capital market: the share of private capital a country can attract depends on the public investment of other countries while the interest
rate depends on their public debt.

Policies depend only on current states $s_{i,t-1}, B_{i,t}$. With identical initial conditions, a symmetric equilibrium can be supported as public spending ultimately depends on the production function parameters, which are identical. Hence, in equilibrium public spending and the capital stock are equal across countries so $g_{i,t} = 1/n$ and $K_{i,t} = K_t/n$.

**Definition 1.** Consider the case of strategic policies with $n$ symmetric countries characterized by $x_i = \{s_{i,t-1}, B_{i,t}\}$. A Markov perfect equilibrium path of this economy is a $n$-tuple of public policy sequences $\{\Theta_{i,t}^s(x_i)\}_{t=0}^{\infty}$ for all $i$ such that $\forall t > 1$ national governments choose $\Theta_{i,t}^s$ to solve (21) given the optimal choices by households and firms, correctly anticipating the future equilibrium policies in all other countries and given other governments’ current policies solve (21).

Note that under strategic policies, while each government takes as given the other governments’ policies, budget constraints hold automatically. This is because factor prices (9) and (10) adjust to maintain the common market general equilibrium.

With logarithmic utility and Cobb-Douglas production function, functional dependence on future policies can be analytically solved for. This enables a two step solution technique, similar to Klein et al. (2008) and Bonatti and Cristini (2008). First, assuming a finite horizon problem, $t = 1, 2, ..., T$, the solution is found solving backwards. Clearly, this implies that governments at $t$ anticipate their effects on $t+1$ governments. Second, iterating on this solution and letting $T \to \infty$ yields the time invariant policy rules for the infinite horizon case. Detailed derivations are relegated to Appendix A.

Given current aggregate capital $K_t = S_{t-1} - B_t$, symmetry implies $K_{i,t}^s = K_t^s/n$. Strategic policy rules $\Theta_{i,t}^s$, next period capital stock $K_{i,t+1}^s$ and the public budget shadow
price $\mu_{i,t}$ are given respectively, by:

$$
\tau^L_{i,t} = 1 - \frac{(1 + \beta)\chi}{z(1 - \sigma)(1 - \alpha)} D^s, \quad (23)
$$

$$
\tau^K_{i,t} = 1 - \frac{(1 - \chi)}{z(1 - \sigma)\alpha} D^s \frac{s_{i,t-1} - B_{i,t}^s}{s_{i,t-1}}, \quad (24)
$$

$$
G_{i,t}^s = (c^s)^\frac{1}{1-\eta} (K_{i,t}^s)^{\frac{\phi}{1-\eta}}, \quad (25)
$$

$$
B_{i,t+1}^s = \frac{c^s + (1 - \sigma)z \left( \frac{1 - \alpha}{1 + \beta} \left( \frac{(1 - \eta)n}{\phi \chi} - 1 \right) - \alpha \right)}{1 + \frac{1 - \eta}{\beta \phi \chi n}} (c^s)^\frac{\eta}{1-\eta} (K_{i,t}^s)^{\frac{\phi}{1-\eta}}, \quad (26)
$$

$$
K_{i,t+1}^s = \chi \beta \phi \frac{D^s}{1 - \eta} \frac{n}{n} (c^s)^\frac{\eta}{1-\eta} (K_{i,t}^s)^{\frac{\phi}{1-\eta}}, \quad (27)
$$

$$
\mu_{i,t}^s = (D^s)^{-1} (c^s)^{-\frac{\eta}{1-\eta}} (K_{i,t}^s)^{-\frac{\phi}{1-\eta}}, \quad (28)
$$

where $c^s$ and $D^s$ are constants given by:

$$
c^s = (1 - \sigma)z \eta \left( \frac{1 - \alpha}{1 - \phi} \left( 1 - \frac{\phi}{n} \right) + \frac{\alpha}{n} \right)
$$

and

$$
D^s = \frac{(1 - \eta) (z(1 - \sigma) - c^s)}{(1 - \eta) + \chi \beta \phi / n}.
$$

Assumption 1 ensures overall decreasing returns to scale and thus a unique and stable steady state, that can be solved for by setting $K_{i,t+1}^s = K_{i,t}^s$ in (27). The steady state capital stock in country $i$ under strategic policies is then:

$$
K_{i,ss}^s = (c^s)^\frac{1}{1-\eta} \left( \frac{\chi \beta \phi D^s}{n(1 - \eta)} \right)^{\frac{1 - \eta}{1-\eta - \sigma}}. \quad (29)
$$

The term $c^s$ captures the externality stemming from competition in public spending. Assumption 1 ensures $D^s$ is positive. On the other hand, higher public spending increases the tax base today and thus reduces the shadow price of the public budget $\mu_{i,t}^s$. On the other hand, the overall cost of funding the public budget is captured by the inverse of the term $D^s$, which depends negatively on $c^s$ since funding public spending is distortionary.
Thus, a higher $D^s$ lowers $\mu_{t,t}^s$ as well as the tax rates and increases the capital stock next period $K_{t,t+1}^s$. At a given level of public spending, $D^s$ increases with $n$ as the pecuniary interest rate externality lowers the cost of debt and thus reduces tax distortions. Note however that future capital depends on $D^s/n$ therefore the public debt externality also has a direct crowding out effect on capital accumulation.

To better understand the role played by each externality in this economy, it is useful to first contrast strategic policies against those resulting from coordination among national governments.

### 3.2 Coordinated policies

Coordinated policies, superscripted $c$, are chosen by a planner that takes into account the welfare of the living agents in all countries as well as all public budget constraints. This policy regime implicitly assumes the existence of a credible enforcement mechanism that would prevent national governments to deviate. Formally, the planner solves the following problem:

$$V^c_t = \max_{\Theta_{j,t}} \sum_{j=1}^{n} U_{j,t}$$

s.t. $B_{j,t+1} + \tau_{j,t}^L w_{j,t} + \tau_{j,t}^K R_t s_{j,t-1} - T_j = G_{j,t} + R_{j,t} B_{j,t}, j = \{1, n\}$. (31)

Coordination takes into account the effects of both domestic public spending and public debt on the other countries in the economy. Note however that here coordination is short-sighted, a consequence of the limited life-span of the households whose preferences are aggregated through the political process. Coordination solves both the fiscal free-riding (through debt) and the fiscal competition (through public spending), but the long run effects of these policies are only partially internalized relative to an infinitely lived planner.

**Definition 2.** Consider the case of coordinated policies with $n$ symmetric countries char-
acterized by \(x_t = \{s_{i,t-1}, B_t\}\). An equilibrium path of this economy is a n-tuple of public policy sequences \(\{\Theta^c_{i,v}(x_v)\}_{v=t}^{\infty}\) for all \(i\) such that \(\forall t > 1\), allocations solve (30) subject to (31), given the optimal choices by households and firms and correctly anticipating that future equilibrium policies solve (30).

Solving for the coordinated policy sequences \(\Theta^c_{i,t}\) (see Appendix A for details) yields:

\[
\tau^L_{i,t} = 1 - \frac{(1 + \beta)\chi}{z(1 - \sigma)(1 - \alpha)} D^c, \tag{32}
\]

\[
\tau^K_{i,t} = 1 - \frac{(1 - \chi)}{z(1 - \sigma)\alpha} D^c \frac{s_{i,t-1} - B^c_{i,t}}{s_{i,t-1}}, \tag{33}
\]

\[
C^c_{i,t} = (c^c)^{\frac{1}{1-\eta}} (K^c_{i,t})^{\frac{\phi}{1-\eta}}, \tag{34}
\]

\[
B^c_{i,t+1} = \frac{c^c + (1 - \sigma)z}{1 + \frac{1-\eta}{\phi\chi}} \left(\frac{(1-\eta)}{\sigma\chi} - 1\right) \left(c^c\right)^{\frac{1-\eta}{1-\eta}} (K^c_{i,t})^{\frac{\phi}{1-\eta}}, \tag{35}
\]

\[
K^c_{i,t+1} = \frac{\chi\beta\phi}{1 - \eta} c^c (c^c)^{\frac{1-\eta}{1-\eta}} (K^c_{i,t})^{\frac{\phi}{1-\eta}}, \tag{36}
\]

\[
\mu^c_{i,t} = (D^c)^{-1} (c^c)^{-\frac{1-\eta}{1-\eta}} (K^c_{i,t})^{-\frac{\phi}{1-\eta}}, \tag{37}
\]

where

\[
c^c = (1 - \sigma)z\eta,
\]

and

\[
D^c = \frac{(1 - \eta)(z(1 - \sigma) - c^c)}{(1 - \eta) + \chi\beta\phi}.
\]

Again, Assumption 1 guarantees overall decreasing returns to scale and the existence of a unique and stable steady state. Under coordination, the steady state aggregate capital stock is:

\[
K^c_{i,ss} = (c^c)^{\frac{1-\eta}{1-\eta-\phi}} \left(\frac{\chi\beta\phi D^c}{1 - \eta}\right)^{\frac{1-\eta}{1-\eta-\phi}}. \tag{38}
\]

Analyzing the coordinated allocations, it is clear that they do not depend on \(n\), the scale of the capital market. In fact, coordinated policies can be obtained by setting \(n = 1\)
in the equations that describe strategic allocations. Thus, $c^c < c^s$ as the strategic motive behind public spending is removed. $D^c > D^s$, i.e. the overall distortion from funding the public budget is lower under coordination due to both reduced financing needs ($G^c_{i,t} < G^s_{i,t}$) and to internalizing crowding out effects. The next section proceeds to compare strategic and coordinated policies more in depth. In particular, I contrast the two sets of policies in the short run, i.e. conditional on the current capital stock, as well as in the steady state.

4 Strategic vs coordinated policies in the short and the long run

Assuming the $n$-country economy is characterized at time $t$ by $s_{i,t-1}, B_{it}$, equations (23)-(27) and (32)-(36) describe the strategic and respectively the coordinated policies. Comparing the two sets of allocations, the following results can be established:

**Proposition 1. Policy comparison in the short run.** Relative to coordinated policies, strategic policies imply:

a) higher public spending $G^s_{i,t} > G^c_{i,t}$,

b) higher share of public debt in output $B^s_{i,t-1}/Y^s_{i,t} > B^c_{i,t-1}/Y^c_{i,t}$,

c) lower tax rates on labor and capital $\tau^L_{i,t} < \tau^L_{i,t}^c$ and $\tau^K_{i,t} < \tau^K_{i,t}^c$,

d) lower capital stock $K^s_{t+1} < K^c_{t+1}$.

Proof. See Appendix B.

International capital mobility leads to overinvestment in public goods but also to a higher public debt share in output. Importantly, while the framework does not feature explicit tax competition, strategic tax rates are nonetheless lower than coordinated ones despite increased needs to fund public spending in order to attract capital. This arises since i) the higher level of productive public spending increases output and thus current tax base and ii) the perceived cost of public debt is too low (relative to coordination) and
thus governments are willing to fund strategic public spending through public debt relative to taxation, whose cost is fully internalized. While the public spending externality has a positive effect on capital accumulation, strategic public debt has a larger negative effect on capital.

Given capital accumulation depends on the policy regime, comparing steady state outcomes becomes a natural next step. Long run policies can be found by substituting steady state capital stocks (29) and (36) in equations (23)-(27) and (32)-(36) respectively. Comparing the two sets of policies, the following results can be established:

**Proposition 2. Policy comparison in the steady state.** Relative to coordinated policies, strategic policies imply:

a) lower public spending: \( G_{s,ss} < G_{c,ss} \).

b) higher share of public debt in output: \( B_{s,ss}/Y_{s,ss} > B_{c,ss}/Y_{c,ss} \).

c) lower tax rates on labor, \( \tau_{L,s,ss} < \tau_{L,c,ss} \), but higher capital tax rates: \( \tau_{K,s,ss} > \tau_{K,c,ss} \).

d) lower capital stock \( K_{s,ss} < K_{c,ss} \).

**Proof.** See Appendix B.

In stark contrast to Proposition 1, in the long run, it is policy coordination that delivers the lower capital tax rates, as well as the higher level of public spending. Given fiscal competition under strategic policies occurs in the form of a "race to the top" in public spending, these outcomes may seem puzzling at first. However, they can be easily understood, once capital accumulation is taken into account. While fiscal competition delivers higher public spending in the short run, this is not enough to compensate for the larger crowding out of private capital due to public debt. As capital accumulation slows down, servicing public debt requires higher tax revenues and at the same time reduces the incentives to engage in public spending competition which further slows down capital accumulation. Steady state interest rates are higher under strategic policies, while wages are lower. This explains why in the long run capital tax rates are higher and labor tax rates are lower under fiscal
competition relative to coordinated policies.

Next, I study how strategic policies are shaped by changes in the scale of financial liberalization, i.e. the number of countries participating in the common capital market.

5 An increase in the scale of financial liberalization (n)

Under strategic policies, capital accumulation depends on the number of countries, n. As changes in the scale of financial liberalization involve a transition towards a new steady state, I first focus on the short run effects of an increase in n. The higher the number of countries, the more intense the fiscal competition in public spending, i.e. \( \partial G^s_{i,t}/\partial n > 0 \).

This follows from \( \partial c^s/\partial n = z\eta(1-\sigma)(\phi-\alpha)/(n^2(1-\phi)) > 0 \) and \( \phi = \alpha(1-\sigma)/(1-2\sigma) > \alpha \).

Thus, ceteris paribus, the larger the output and the capital stock next period. The effects on the term \( D^s \) are more involved. On the one hand, higher n increases \( D^s \) through the term in the denominator. This stems from the pecuniary interest rate externality that leads governments to underprice their debt. On the other hand, higher public spending lowers \( D^s \) as \( c^s \) is increasing in n and thus magnifies tax distortions.

**Proposition 3.** In the short run, an increase in scale of financial integration (the number of countries n) leads to:

a) higher public spending \( \partial G^s_{i,t}/\partial n > 0 \),

b) higher share of public debt in output: \( \partial (B^s_{i,t+1}/Y^s_{i,t})/\partial n > 0 \),

c) lower tax rates: \( \partial L^s_{i,t}/\partial n < 0 \) and \( \partial K^s_{i,t}/\partial n < 0 \) if \( \beta\gamma > (1-\gamma)\eta(\phi-\alpha)/\eta(1-\gamma-\phi+\alpha\gamma) \),

d) lower capital stock: \( \partial K^s_{i,t+1}/\partial n < 0 \).

**Proof.** See Appendix B. \( \square \)

More intense fiscal competition implies that even as they spend more, governments issue more debt and thus can set lower tax rates. The condition under which tax rates decline
with \( n \) is related to the conflict of interests between young and old agents. Intuitively, higher \( \beta \) (the discount rate) or \( \chi \) (the political weight of young agents) imply national governments place a larger weight on the lifetime welfare of the current young. In order to lower the capital tax rates on their saving, they increase current public debt whose burden will be shared with the future young generation. Capital accumulation is hindered as the crowding out due to public debt dominates the crowding in effects due to public spending.

Since coordinated policies replicate the one country case, Proposition 2 can in fact be used to predict the steady state impact of an increase in \( n \). Thus, one can expect higher steady state capital tax rates, higher debt but lower public spending and capital.

So far, I have compared strategic and coordinated policies taking each regime as given. In the following, I assume both regimes can be implemented at time \( t \) and study the choice of national governments by comparing the social welfare levels associated with each regime, both in the short run and in the steady state.

6 Welfare analysis

The welfare of the currently living generations in country \( i \) can be expressed for each policy regime \( x = \{s,c\} \), in terms of the shadow prices associated with the government budget constraints at \( t \) and \( t+1 \):

\[
V_{i,t}^x = (1 - \chi) \ln \left( \frac{1 - \chi}{\mu_{i,t}^x} \right) + \chi \ln \left( \frac{1 + \beta \chi}{\mu_{i,t+1}^x} \right) + \beta \chi \ln \left( \frac{1 - \chi}{\mu_{i,t+1}^x} \right),
\]

where \( \chi \) is the weight of the young generation in the social welfare function.

Let \( \Omega_t = V_{i,t}^s - V_{i,t}^c \) denote the welfare difference between strategic and coordinated policies, or equivalently:

\[
\Omega_t = \ln \left( \frac{\mu_{i,t}^c}{\mu_{i,t}^s} \right) \left( \frac{\mu_{i,t+1}^c}{\mu_{i,t+1}^s} \right)^{\chi \beta}.
\]

Intuitively, expression (40) links the welfare differential to the relative cost (across the
two policy regimes) of funding the public budget today (this affects both young and old agents, so it has a weight of one), and tomorrow (this affects only the currently young agents, so it has a discounted weight of $\chi \beta$). Using the steady state capital stocks (29) and (38) in the expressions for $\mu_{i,t}^s$ and $\mu_{i,t}^c$, (28) and (37) respectively, yields the steady state shadow prices $\mu_{i,ss}^s$, $\mu_{i,ss}^c$ and the corresponding long run welfare differential $\Omega_{ss} = \ln \left( \frac{\mu_{i,ss}^c}{\mu_{i,ss}^s} \right)^{1+\chi \beta}$.

Signing the welfare difference on a transition ($\Omega_t$) and in the steady state ($\Omega_{ss}$) yields the following results:

**Proposition 4.** On a transition path, strategic policies yield higher welfare for the currently living generations relative to coordinated policies only if the number of countries is lower than some threshold $\bar{n}$. However, in the steady state, coordinated fiscal policies deliver the highest welfare, $\forall n$.

**Proof.** See Appendix B.  

The proposition summarizes an important result: in a world where governments use deficit funded public spending to compete for mobile capital, policy coordination can be inferior in the short run despite being always optimal in the long run. Importantly, while the assumption of short-sighted governments facilitates a tractable analysis, the result hinges on the existence of two distinct cross-border fiscal externalities that arise regardless of the governments’ time horizon, depending, in turn, on the number of countries sharing the capital market.

To understand why fiscal competition prevails if the scale of the global capital market is too low, recall that while both public spending and debt have external effects, the induced externalities change at different rates with the number of countries. The externality from productive public spending leads to higher current incomes and thus welfare. While this externality dominates initially, uncoordinated public debt increases faster with $n$. Eventually, crowding out of private capital becomes large enough to warrant a switch to
coordinated policies even by short lived governments.

7 Concluding remarks

The paper develops a dynamic theory of fiscal competition via both productive public spending and debt. In contrast to tax competition models, in this framework residence based taxation is feasible. Nonetheless, in the short run fiscal competition lowers capital taxation relative to coordination. The result is driven here by a totally different mechanism, namely political-economy rooted myopia coupled with the possibility to increase the tax base by funding productive public spending with debt. However, long run capital tax rates are higher under fiscal competition which sets the world economy on a path of lower capital accumulation and thus higher interest rates.

Following an increase in the scale of financial liberalization, modelled as in increase in the number of countries that participate in the common capital market, capital tax rates can decline despite higher public spending as public debt rises in all countries disproportionately. This is relevant in the context of the long-standing debate on harmful tax competition and the recently implemented measures aimed at facilitating residence based taxation. These measures are expected to prevent a "race to the bottom" in capital tax rates and thus ensure more public budget revenues. In contrast to conventional wisdom, this paper shows capital tax rates may continue to decline in the short run as countries engage in deficit spending.

The paper also sheds new light on the dynamic welfare effects of fiscal competition. In the steady state, coordinated policies dominate unambiguously such that even short sighted governments implement coordination. In contrast, along a transition path, the scale of financial integration is critical: coordinated policies generate higher welfare for the currently living generations only if the number of countries sharing the common capital market is large enough.
To keep the analysis tractable, the model has been simplified along a number of dimensions. First, while fiscal free riding through public debt implies a form of tax competition as governments are able to cut current tax rates and pass the costs onto the other regions, direct tax competition is not considered. Including this channel as well as additional spending outlays, such as consumption public goods, public education or intergenerational transfers would not alter qualitatively the externalities that underpin the main results. While the paper focuses on symmetric equilibria, coordination may be harder to implement between heterogenous countries. Indeed, Kanbur and Keen (1993) find in a static model that despite added inefficiencies from size differences tax harmonization may still be suboptimal. Also, the model abstracts from other linkages, such as labor mobility, trade flows or monetary policy that may limit/amplify fiscal free riding. All these avenues are left for future research.

References


Appendix A  Equilibrium policy functions

Strategic policies:

\[
\max_{\Theta_{i,t}} \{ U_{i,t} + \mu_{i,t} \left[ B_{i,t+1} - R_{i,t}B_{i,t} - G_{i,t} + w_{i,t}\tau_{i,t}^I + s_{i,t-1}R_{i,t}\tau_{i,t}^K \right] \}
\]

The solution of the non-cooperative game is found solving the game backwards. Assume a terminal period of the economy \( T \). The economy is characterized by the aggregate stock of capital \( K_T \) and the savings and bonds in each country: \( s_{i,T-1}, B_{i,T} \). Since \( T \) is assumed to be the last period of the economy, no bonds are issued hence \( B_{i,T+1} = 0 \) and young households consume their entire income, so \( s_{i,T} = 0 \). Taxes in \( T \) are set to finance the repayment of outstanding debt, \( R_T B_{i,T} \) and current public spending \( G_{i,T} \).

Public policies are linked through the contemporaneous capital market, described by (15). Since the old age welfare of the agents that are young at \( T \) does not matter, the government’s problem in country \( i \) is linked to the choices of the other governments only through the current fiscal competition in public spending.

Consumption flows at \( T \) are:

\[
c_{i,T}^0 = w_{i,T}(1 - \tau_{i,T}^L) = \alpha Y_T(1 - \tau_{i,T}^L); c_T^0 = s_{i,T-1}R_T(1 - \tau_{i,T}^K).
\]

The government maximizes:

\[
\max_{G_{i,T}, \tau_{i,T}^L, \tau_{i,T}^K} \{ \chi \ln[w_{i,T}(1 - \tau_{i,T}^L)] + (1 - \chi) \ln[s_{i,T-1}R_T(1 - \tau_{i,T}^K)] + \mu_{i,T} \left[ -R_T B_{i,T} - G_{i,T} + w_{i,T}\tau_{i,T}^I + s_{i,T-1}R_T\tau_{i,T}^K \right] \},
\]

\[^{11}\text{See Klein et al (2008) refered in the main text for a similar solution technique.}\]
given the state variables \( \{K_T, B_i, s_i, T-1\} \), \( i = \{1, 2, \ldots, n\} \) and policies chosen by other governments. Finally, \( \mu_{i,T} \) is the Lagrange multiplier associated with the budget constraint of country \( i \).

Taking the first order conditions yields:

\[
\tau^K_{i,T} : -\frac{\chi}{1 - \tau^K_{i,T}} + \mu_{i,T} s_{i,T-1} R_T = 0, \\
\tau^L_{i,T} : -\frac{\chi}{1 - \tau^L_{i,T}} + \mu_{i,T} w_{i,T} = 0, \\
G_{i,T} : \left( \frac{\chi}{w_{i,T}} + \mu_{i,T} \tau^L_{i,T} \right) \frac{\partial w_{i,T}}{\partial G_{i,T}} + \left( 1 - \frac{\chi}{R_T} - \mu_{i,T} B_{i,T} + \mu_{i,T} s_{i,T-1} R^K_{i,T} \right) \frac{\partial R_T}{\partial G_{i,T}} - \mu_{i,T} = 0.
\]

Expression (17) can be used to rewrite prices to reflect the inter-dependency between national policy choices:

\[
w_{i,T} = (1 - \sigma)(1 - \alpha) Y_{i,T} = (1 - \sigma)(1 - \alpha) z G_{i,T}^{-1} \left( \sum_{j=1}^{n} G_{j,T}^{\eta} \right)^{-\phi} K_T^{\phi},
\]

\[
q_T = R_T = (1 - \sigma) \alpha K_{i,T} = (1 - \sigma) \alpha z \left( \sum_{j=1}^{n} G_{j,T}^{\eta} \right)^{1-\phi} K_T^{-1+\phi}.
\]

Using these expressions to compute the marginal effect of domestic public spending yields:

\[
\frac{\partial w_{i,T}}{\partial G_{i,T}} = \frac{(1 - \sigma)(1 - \alpha) z \eta}{1 - \phi} \frac{G_{i,T}^{\eta - 1} \left( \sum_{j=1}^{n} G_{j,T}^{\eta} - \phi G_{i,T}^{\eta} \right)}{\left( \sum_{j=1}^{n} G_{j,T}^{\eta} \right)^{\phi+1}} K_T^{\phi},
\]

\[
\frac{\partial R_T}{\partial G_{i,T}} = (1 - \sigma) \alpha z \eta \frac{G_{i,T}^{\eta - 1} \left( \sum_{j=1}^{n} G_{j,T}^{\eta} \right)^{-\phi}}{K_T} K_T^{\phi}.
\]

Under symmetry (A.6), (A.7) and (A.24) become:

\[
\frac{\partial w_{i,T}}{\partial G_{i,T}} = \frac{(1 - \sigma)(1 - \alpha) z \eta}{1 - \phi} \left( \frac{K_T}{n} \right)^{\phi} G_{i,T}^{\eta - 1} \left( 1 - \phi \frac{n}{1 - \phi} \right),
\]

\[
\frac{\partial R_T}{\partial G_{i,T}} = (1 - \sigma) \alpha z \eta \frac{1}{K_T} \left( \frac{K_T}{n} \right)^{\phi} G_{i,T}^{\eta - 1}.
\]

26
and \( K_{i,T} = K_T/n \), \( s_{i,T-1} = S_{T-1}/n \) and \( B_{i,T} = B_T/n \). Using these expressions in the first order conditions (A.1)-(A.3), together with the capital market clearing condition (15) yields the optimal policies at \( T \):

\[
\tau^L_{i,T} = 1 - \frac{\chi (z(1-\sigma) - c^s)}{z(1-\alpha)(1-\sigma)},
\]

\[
\tau^K_{i,T} = 1 - \frac{(1-\chi) (z(1-\sigma) - c^s) s_{i,T-1} - B_{i,T}}{z\alpha(1-\sigma)}
\]

\[
G_{i,T} = (c^s)^{\frac{1}{1-\eta}} \left( \frac{K_T}{n} \right)^{\frac{z\alpha}{1-\eta}}
\]

where

\[
c^s = (1-\sigma)z\left( \frac{1-\alpha}{1-\phi} \left( 1 - \frac{\phi}{n} \right) + \frac{\alpha}{n} \right).
\]

Then, \( Y_{i,T} = z (c^s)^{\frac{n}{1-\eta}} \left( \frac{K_T}{n} \right)^{\frac{z\alpha}{1-\eta}} \). Using the above allocations in the government budget constraint yields the shadow value of relaxing the government budget constraint at \( T \):

\[
\mu_{i,T} = 1/(Y_{i,T}(1-\sigma) - G_{i,T}).
\]

At time \( T-1 \), the government takes as given the optimal policy rules in \( T \) (i.e. anticipates the reaction of next period government to current policies) and the state of the economy at \( T-1 \) given by \( \{K_{T-1}, s_{i,T-2}, B_{i,T-1}\} \). Now, the maximization problem includes the old-age welfare of the agents that are young at \( T \).

\[
\max_{\tau^K_{i,T-1}, \tau^K_{i,T-1}, G_{i,T-1}, B_{i,T}} \left\{ \chi \ln c^p_{i,T-1} + \chi \beta \ln c^p_{i,T} + (1-\chi) \ln c^p_{i,T-1} \right\}
\]

Again, prices are given by expressions (6) and (7) with the time index adjusted properly. Moreover, countries interact through the aggregate capital stock \( K_T \):

\[
K_T = \sum_{i=1}^{n} (s_{i,T-1} - B_{i,T}).
\]

As opposed to the fiscal competition channel, this is a dynamic externality, due to capital accumulation. Young agents in period \( T-1 \) save for the old age and their welfare at \( T \) depends on the interest rate \( R_T \) which in turn depends on the (strategic) policies implemented at \( T \) and \( T-1 \) in all countries. The private policy function for savings is:

\[
s_{i,T-1} = \frac{\beta}{1+\beta}(1-\sigma)(1-\alpha)Y_{i,T-1}
\]

Also, using the terminal period capital tax policy in the utility of the old agents at \( T \)
yields their consumption flow anticipated at $T - 1$:

$$s_{i,T-1} R_T (1 - \tau^K_{i,T-1}) = (1 - \chi)/\mu_{i,T}.$$ 

Thus, the marginal change in the future welfare of this group from a change in public spending at $T - 1$ is:

$$\frac{\partial (\chi \beta \ln \mu_{i,T})}{\partial G_{i,T-1}} = \frac{\chi \beta}{\mu_{i,T}} \frac{\partial K_T}{\partial s_{i,T-1}} \frac{\partial w_{i,T-1}}{\partial w_{i,T-1}} \frac{\partial s_{i,T-1}}{\partial G_{i,T-1}} = \frac{\chi \beta}{\mu_{i,T}} \left( -\frac{\phi}{1 - \eta} \right) \mu_{i,T} \frac{\beta}{K_T} \frac{1}{1 + \beta} \frac{\partial w_{i,T-1}}{\partial G_{i,T-1}}.$$ 

The corresponding change from an extra unit of public debt issued at $T - 1$ is:

$$\frac{\partial (\chi \beta \ln (1/\mu_{i,T}))}{\partial B_{i,T}} = -\frac{\chi \beta}{\mu_{i,T}} \frac{\partial K_T}{\partial G_{i,T-1}} \frac{\partial s_{i,T-1}}{\partial G_{i,T-1}} = -\frac{\chi \beta}{\mu_{i,T}} \frac{\phi}{1 - \eta} \frac{\mu_{i,T}}{K_T}.$$ 

Substituting households allocations (2) - (4), prices (6) and (7), and the optimal policies at $T$ in (A.12) results in the Lagrangian:

$$\max_{T_{i,T-1}^L, T_{i,T-1}^K, G_{i,T-1}, B_{i,T}} \{ \chi \ln[(w_{i,T-1} - s_{i,T-1})(1 - \tau^L_{i,T-1})] + (1 - \chi) \ln[s_{i,T-2} R_{T-1} (1 - \tau^K_{i,T-1})] + \chi \beta \ln \left[ s_{i,T-1} R_T (1 - \tau^K_{i,T}) \right] + \mu_{i,T-1} \left[ B_{i,T} - R_{T-1} b_{i,T-1} - G_{i,T-1} + (w_{i,T-1} - s_{i,T-2}) \tau^L_{i,T-1} + s_{i,T-2} R_{T-1} \tau^K_{i,T-1} \right] \},$$

The first order conditions are given by:

$$T_{i,T-1}^L : -\frac{\chi}{1 - \tau^L_{i,T-1}} + \frac{\mu_{i,T-1}}{1 + \beta} w_{i,T-1} = 0,$$  

$$T_{i,T-1}^K : \frac{1}{1 - \tau^K_{i,T-1}} + \mu_{i,T-1} s_{i,T-2} R_{T-1} = 0,$$ 

$$G_{i,T-1} : \left( \frac{\chi}{w_{i,T-1}} + \frac{\mu_{i,T-1} - \tau^L_{i,T-1}}{1 + \beta} \right) \frac{\partial w_{i,T-1}}{\partial G_{i,T-1}} + \left( \frac{1 - \chi}{R_{T-1}} - \mu_{i,T-1} B_{i,T-1} + \mu_{i,T-1} s_{i,T-2} \tau^K_{i,T-1} \right) \frac{\partial R_{T-1}}{\partial G_{i,T-1}} + \frac{\chi \beta}{\mu_{i,T}} \frac{\partial \mu_{i,T}}{\partial G_{i,T-1}} = 0,$$ 

$$B_{i,T} : -\frac{\chi \beta}{\mu_{i,T}} \frac{\partial \mu_{i,T}}{\partial B_{i,T}} + \mu_{i,T-1} = 0.$$
Imposing symmetry of the states \(\{s_{i,T-2}, B_{i,T-1}\}\) and using (A.13)-(A.16) together with the budget constraint yields:

\[
G_{i,T-1} = (c^s)^{\frac{1}{1-\eta}} \left( \frac{K_{T-1}}{n} \right)^{\frac{\phi}{1-\eta}},
\]

\[
\tau^L_{i,T-1} = 1 - \frac{\chi(1+\beta)}{z(1-\alpha)(1-\sigma)} D^s,
\]

\[
\tau^K_{i,T-1} = 1 - \frac{(1-\chi)}{z\alpha(1-\sigma)} D^s \frac{s_{i,T-2} - B_{i,T-1}}{s_{i,T-2}},
\]

where \(c^s\) has been defined above,

\[
D^s = \frac{(1-\eta)(z(1-\sigma) - c^s)}{1-\eta + \chi\beta\phi/n},
\]

and \(Y_{i,T-1} = z (c^s)^{\frac{n}{1-\eta}} \left( \frac{K_{T-1}}{n} \right)^{\frac{\phi}{1-\eta}}\).

\[
K_{i,T} = \frac{\beta\chi\phi}{1-\eta} \frac{D^s}{n} (c^s)^{\frac{n}{1-\eta}} \left( \frac{K_{T-1}}{n} \right)^{\frac{\phi}{1-\eta}},
\]

(A.17)

\[
s_{i,T-1} = \frac{z\beta}{1+\beta} (1-\sigma)(1-\alpha) (c^s)^{\frac{n}{1-\eta}} \left( \frac{K_{T-1}}{n} \right)^{\frac{\phi}{1-\eta}},
\]

(A.18)

\[
B_{i,T} = \frac{c^s + (1-\sigma) z }{1 + \frac{1}{z\beta\phi n}} \frac{1-\alpha}{(1-\eta)n} \left( \frac{K_{T-1}}{n} \right)^{\frac{\phi}{1-\eta}}.
\]

(A.19)

\[
\mu_{i,T-1} = (c^s)^{\frac{n}{1-\eta}} \left( \frac{K_{T-1}}{n} \right)^{\frac{\phi}{1-\eta}} (D^s)^{-1}.
\]

(A.20)

Substituting these time invariant allocations (A.17), (A.18), (A.19) and (A.20) in the set of first order conditions, one gets the equilibrium policies (19) - (22). These policies support a symmetric equilibrium given identical initial conditions. Moreover, letting \(T \to \infty\) and using (A.20) repeatedly in periods \(T-j\), where \(j \to \infty\) yields the equilibrium policy functions, the implied capital stock (23) and the shadow price (24) in the infinite horizon setup.

**Coordinated policies:**

Fiscal policies under coordination are derived in a similar manner.

\[
\max_{G_{i,T}, \tau^L_{i,T}, \tau^K_{i,T}} \sum_{i=1}^n \left\{ \chi \ln[w_{i,T}(1 - \tau^L_{i,T})] + (1 - \chi) \ln[s_{i,T-1}R_T(1 - \tau^K_{i,T})] \right. \\
+ \left. \mu_{i,T} \left[ -R_T B_{i,T} - G_{i,T} + w_{i,T} \tau^L_{i,T} + s_{i,T-1}R_T \tau^K_{i,T} \right] \right\}
\]
While first order conditions for tax rates are similar to (A.8) and (A.9), the planner takes into account cross country effects of public spending on the interest rate:

\[ G_{i,T} : \frac{\chi}{w_{i,T}} + \mu_{i,T} \frac{\tau_{i,T}}{w_{i,T}} + \sum_{j=1,j \neq i}^{n} \left( \frac{\chi}{w_{j,T}} + \mu_{j,T} \frac{\tau_{j,T}}{w_{j,T}} \right) \frac{\partial w_{j,T}}{\partial G_{i,T}} + \sum_{j=1}^{n} \left( 1 - \chi \right) \frac{R_{T}}{R_{T}} - \mu_{j,T} B_{j,T} + \mu_{j,T} s_{j,T-1} \frac{\tau_{j,T}}{w_{j,T}} \right) - \frac{\partial R_{T}}{\partial G_{i,T}} - \mu_{i,T} = 0. \]  

(A.21)

Imposing symmetry and solving for \( G_{i,T} \) yields

\[ G_{i,T} = (c^{\phi}) \frac{k_{T}}{\eta-n}, \]

where \( c^{\phi} = (1 - \sigma) \zeta \eta \). Note that \( c^{\phi} = c^{\phi} \) for \( n = 1 \). The coordinated solution mirrors indeed policy choices in a one economy world. At \( T - 1 \) the planner solves:

\[ \max_{\tau_{i,T-1}^{L}, \tau_{i,T-1}^{R}, B_{i,T}} \sum_{i=1}^{n} \left\{ \chi \ln[(w_{i,T-1} - s_{i,T-1})(1 - \tau_{i,T-1}^{L})] + (1 - \chi) \ln[s_{i,T-2} R_{T-1}(1 - \tau_{i,T-1}^{R})] + \chi \beta \ln[s_{i,T-1} R_{T}(1 - \tau_{i,T-1}^{K})] + \mu_{i,T-1} \left[ B_{i,T} - R_{T-1} B_{i,T-1} - G_{i,T-1} + \left( w_{i,T-1} - s_{i,T-2} \right) \tau_{i,T-1}^{L} + s_{i,T-2} R_{T-1} \tau_{i,T-1}^{K} \right] \right\}. \]

For any \( t < T \) first order conditions for tax rates are similar to (A.13) and (A.13) and the planner takes into account cross country effects of both national public spending and debt:

\[ G_{i,T-1} : \frac{\chi}{w_{i,T-1}} + \mu_{i,T-1} - \tau_{i,T-1}^{L} \frac{\partial w_{i,T-1}}{\partial G_{i,T-1}} + \sum_{j=1,j \neq i}^{n} \left( \frac{\chi}{w_{j,T-1}} + \mu_{j,T-1} \frac{\tau_{j,T-1}^{L}}{w_{j,T-1}} \right) \frac{\partial w_{j,T-1}}{\partial G_{i,T-1}} + \sum_{j=1}^{n} \left( 1 - \chi \right) \frac{R_{T-1}}{R_{T-1}} - \mu_{j,T-1} B_{j,T} + \mu_{j,T-1} s_{j,T-2} \frac{\tau_{j,T-1}^{K}}{w_{j,T-1}} \right) \frac{\partial R_{T-1}}{\partial G_{i,T-1}} + \chi \beta \sum_{j=1}^{n} \frac{1}{\mu_{j,T-1}} \frac{\partial \mu_{j,T-1}}{\partial G_{i,T-1}} - \mu_{i,T-1} = 0. \]  

(A.22)

\[ B_{i,T} : -\chi \beta \sum_{j=1}^{n} \frac{1}{\mu_{j,T}} \frac{\partial \mu_{j,T}}{\partial B_{i,T}} + \mu_{i,T} = 0. \]  

(A.23)

where the effect of public debt on the cost of future public resources in all countries is internalized as is the effect of the public spending on foreign countries:

\[ \frac{\partial w_{i,T}}{\partial G_{j,T}} = -\frac{\phi}{1 - \phi} x(1 - \alpha)(1 - \alpha) G_{i,T}^{\frac{n\phi}{1-\phi}} \left( \sum_{j=1}^{n} G_{j,T}^{\frac{n\phi}{1-\phi}} \right)^{\phi+1} G_{j,T}^{\frac{n\phi}{1-\phi}-1} \tau_{j,T}^{\phi}. \]  

(A.24)

Following similar steps as in the case of strategic policies yields the equilibrium policy
functions (32)-(36).

Appendix B  Proofs

Proof of Proposition 1.

a) $c^s > c^c \Rightarrow G^i_{i,t} > G^c_{i,t}, \forall n > 1$.  

b) See part b) of Proposition 3. Note that $f^b(n) = B^s_{i,t+1}/Y^s_{i,t}$ and $f^b(1) = B^c_{i,t+1}/Y^c_{i,t}$.  

Thus, since $\partial f^b/\partial n > 0, \forall n > 1, B^s_{i,t+1}/Y^s_{i,t} > B^c_{i,t+1}/Y^c_{i,t}$.

c) Follows from $\frac{1-\tau^L_{i,t}}{1-\tau^C_{i,t}} = \frac{c^s - z(1-\alpha)}{c^c - z(1-\alpha)} < 1$ for $n > 1$ and $\delta$ small, since $c^s > c^c$. Similarly, $\tau^L_{i,t} < \tau^C_{i,t}$.

d) The result follows from part d) of Proposition 3. Note that $f^k(1) = K^C_{i,t+1}$ and $g^k(1) = K^C_{ss}$. Conditional on current capital stock $K^s_{i,t+1}/K^c_{i,t+1} < 1 \Leftrightarrow K^s_{ss}/K^c_{ss} < 1$.

Proof of Proposition 2.

a) Using the definitions of $G^s_{i,ss}$ and $G^c_{i,ss}$ together with those for the steady state capital stocks $K^s_{i,ss}$ and $K^c_{i,ss}$, for $G^s_{i,ss}/G^c_{i,ss} < 1$ it is sufficient to show:

$$x^{1-\phi} \left( \frac{1 - \eta x}{1 - \eta} \right)^{1-\phi} \left( \frac{1 - \eta + \beta \phi \chi}{n(1 - \eta) + \beta \phi \chi} \right) < 1. \quad (B.1)$$

where $x = 1 + (n - 1)(\phi - \alpha)/(n(1 - \phi)) > 1$. The third parenthesis is lower than one for $n > 1$ and $(1 - \eta x)/(1 - \eta) < 1$.

b) Denote $f^b(n) = B^s_{i,t+1}/Y^s_{i,t} = \frac{c^s - z(1-\alpha)}{c^c - z(1-\alpha)}$. Then

$$\frac{\partial f^b}{\partial n} = \frac{\beta \phi \chi (1 - \sigma) [n(1 - \eta)(n(1 - (1 - \alpha)\eta - \phi) + 2\eta(\phi - \alpha)) + \beta \eta \phi \chi (\phi - \alpha)]}{n^2(1 - \phi)(n(1 - \eta) + \beta \phi \chi)^2} > 0$$

since $\phi + \eta < 1$ and $\alpha < 1 \Rightarrow 1 - (1 - \alpha)\eta - \phi > 0$ and $\phi = \alpha(1 - \sigma)/(1 - 2\sigma) > \alpha$. Note that $f^b(n) = B^s_{i,t+1}/Y^s_{i,t} = B^s_{i,ss}/Y^s_{i,ss}$ and $f^b(1) = B^c_{i,t+1}/Y^c_{i,t} = B^c_{i,ss}/Y^c_{i,ss}$. Thus, since $\partial f^b/\partial n > 0, \forall n > 1, B^s_{i,ss}/Y^s_{i,ss} > B^c_{i,ss}/Y^c_{i,ss}$.

c) Labor taxes do not depend on the capital stock so part c) of Proposition 1 also implies $\tau^L_{i,ss} < \tau^C_{i,ss}$. On the other hand steady state capital taxes depend on capital stocks and saving levels. Setting equations (33) and (24) to the steady state implies, after simplifications:

$$\frac{1 - \tau^L_{i,ss}}{1 - \tau^C_{i,ss}} = \left(1 - \frac{\eta(n - 1)(\phi - \alpha)}{n(1 - \phi)(1 - \eta)}\right)^2 \left(1 - \frac{(n - 1)(1 - \eta)}{n(1 - \phi) + \beta \phi \chi}\right)^2 \frac{1}{n} < 1,$$

which holds $\forall n > 1$ since all the terms in parentheses are positive and subunitary.

Proof of Proposition 3.

a) Follows from $\partial c^s/\partial n = \eta(1-\sigma)(\phi - \alpha)/(n^2(1 - \phi)) > 0$ and $\phi = \alpha(1 - \sigma)/(1 - 2\sigma) > \alpha$.  

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b) Denote \( f^h(n) = B^s_{i,t+1}/Y^s_{i,t} \). Then

\[
\frac{\partial f^h}{\partial n} = \frac{\beta \phi \chi (1-\sigma) [n(1-\eta) (n(1-\alpha) \eta - \phi) + 2\eta(\phi - \alpha))] + \beta \eta \phi \chi (\phi - \alpha)]}{n^2(1-\phi) (n(1-\eta) + \beta \phi \chi)^2} > 0
\]

since \( \phi + \eta < 1 \) and \( \alpha < 1 \Rightarrow 1-(1-\alpha)\eta - \phi > 0 \) and \( \phi = \alpha(1-\sigma)/(1-2\sigma) > \alpha \).

\( \frac{\partial \tau^K_{i,t}}{\partial n} < 0 \Leftrightarrow \partial D^s/\partial n > 0 \Leftrightarrow \frac{2(1-\eta)(1-\sigma)(\beta \phi \chi (\phi - \alpha) - (1-\eta)(\phi - \alpha))}{(1-\phi)(n(1-\eta) + \beta \phi \chi)^2} > 0 \). The latter inequality yields the condition \( \beta \chi > (1-\eta)\eta(\phi - \alpha)/(\phi(1-\eta - \phi + \alpha \eta)) \). A similar condition can be derived for labor tax rates.

d) Denote \( f^k(n) = K^s_{i,t+1}/n \) and \( g^k(n) = K^s_{i,t+1}/n \). Conditional on current capital stock, \( f^k(n) < f^k(1) \Leftrightarrow g^k(n) < g^k(1), \forall n > 1 \). Thus, focusing on the latter inequality, it is sufficient to show that

\[
\left( \frac{c^s}{c^c} \right)^{\eta/(1-\eta)} \frac{z(1-\sigma) - c^s}{z(1-\sigma) - c^s} \frac{1 - \eta + \chi \beta \phi}{n(1-\eta) + \chi \beta \phi} < 1. \tag{B.2}
\]

Substituting the expressions for \( c^s \) and \( c^c \) in (B.2) and simplifying yields:

\[
\left( 1 + \frac{(n-1)(\phi - \alpha)}{n(1-\phi)} \right)^{\eta/(1-\eta)} \left( 1 - \frac{\eta(n-1)(\phi - \alpha)}{n(1-\phi)(1-\eta)} \right) \left( 1 - \frac{(n-1)(1-\eta)}{n(1-\eta) + \beta \phi \chi} \right) < 1
\]

First note \( \eta/(1-\eta) < 1 \) since \( \delta \leq \alpha \) and \( (\alpha + \delta)(1-\sigma)/(1-2\sigma) < 1 \) imply \( \eta < 1/2 \) for the limiting case \( \alpha = \delta \). Then, for \( p, x < 1 \) use \( (1+x)p(1+px) < (1+x)(1-x) < 1 \) for \( p = \eta/(1-\eta) \) and \( x = (n-1)(\phi - \alpha)/n(1-\phi) < 1 \). Finally, \( 1-(n-1)(1-\eta)/(n(1-\eta) + \beta \phi \chi) < 1 \) holds for any \( n > 1 \).

**Proof of Proposition 4.**

The welfare difference at \( t, \Omega_t = V^s_{i,t} - V^c_{i,t} \) is rewritten as:

\[
\Omega_t \equiv \ln \left( \frac{\mu^c_{i,t} \mu^c_{i,t+1}}{\mu^s_{i,t} \mu^s_{i,t+1}} \right)^{\chi \beta}, \tag{B.3}
\]

where

\[
\frac{\mu^c_{i,t+1}}{\mu^s_{i,t+1}} = \left( \frac{\mu^c_{i,t}}{\mu^s_{i,t}} \right)^{1 + \frac{\phi}{n}} n^{-\frac{1+\phi}{n}}, \tag{B.4}
\]

\[
\frac{\mu^c_{i,t}}{\mu^s_{i,t}} = n \left( 1 + \frac{(n-1)(\phi - \alpha)}{n(1-\phi)} \right)^{1-\eta} \left( 1 - \frac{\eta(n-1)(\phi - \alpha)}{n(1-\phi)(1-\eta)} \right) \frac{1 - \eta + \chi \beta \phi}{n(1-\eta) + \chi \beta \phi}. \tag{B.5}
\]

Substituting (B.4) together with (37) in (B.3) and simplifying implies \( \Omega_t < 0 \) is equivalent
to:

\[(1 + \chi\beta)\ln n + \left(1 + \chi\beta \left(1 + \frac{\phi}{1 - \eta}\right)\right) \times \]

\[
\left[\frac{n}{1 - \eta} \ln \left(1 + \frac{n - 1}{n} \frac{\phi - \alpha}{1 - \phi}\right) + \ln \left(1 - \frac{n}{1 - \eta} \frac{n - 1}{n} \frac{\phi - \alpha}{1 - \phi}\right) - \frac{\ln \left(\frac{n(1 - \eta) + \chi \beta \phi}{1 - \eta + \chi \beta \phi}\right)}{1 - \eta + \chi \beta \phi}\right] < 0 \tag{B.6}
\]

Applying \(x \gtrsim \ln(1 + x)\) for small \(x\) to the terms on the second line above yields:

\[(1 + \chi\beta)\ln n < \left(1 + \chi\beta \left(1 + \frac{\phi}{1 - \eta}\right)\right) \ln \left(\frac{n(1 - \eta) + \chi \beta \phi}{1 - \eta + \chi \beta \phi}\right) \tag{B.7}
\]

and using \(\frac{n(n-\eta) + \chi \beta \phi}{1 - \eta + \chi \beta \phi} > \frac{n(n-\eta)}{1 - \eta + \chi \beta \phi}\) leads to the sufficient condition:

\[-\frac{\phi \chi \beta}{1 - \eta} \ln n < \left(1 + \chi\beta \left(1 + \frac{\phi}{1 - \eta}\right)\right) \ln \left(\frac{1 - \eta}{1 - \eta + \chi \beta \phi}\right) \tag{B.8}
\]

This is satisfied for \(n > \frac{\bar{n}}{1 - \eta} = (1 + \chi\beta \phi/1 - \eta) (1 - \eta/1 - \eta) \phi (1 + \chi\beta)/\phi (\chi\beta)\).

**Ranking of steady state welfare levels:** Using capital stocks (29) and (38) in (28) and (37) respectively yields the steady state welfare differential \(\Omega_{ss} = V_{i,ss}^s - V_{i,ss}^c\):

\[
\Omega_{ss} = \ln \left(\frac{\mu_i^{ss}}{\mu_i^{ss}}\right)^{1+\chi\beta} = \ln \left(n^{1+\chi\beta} \left[1 + \frac{(n - 1)(\phi - \alpha)}{n(1 - \phi)}\right]^{\frac{n}{1 - \eta}} \left(1 - \frac{n(n - 1)(\phi - \alpha)}{(1 - \eta) n(1 - \phi)}\right) \frac{1 - \eta + \chi \beta \phi}{n(1 - \eta) + \chi \beta \phi}\right) \tag{B.9}
\]

Coordinated policies yield higher steady state welfare if \(\Omega_{ss} < 0\). Using the log approximation to cancel the first two parentheses, after some simplifications, the inequality is equivalent with:

\[
\frac{\phi}{n(1 - \eta)} \frac{1 - \eta + \chi \beta \phi}{LHS(n)} > \frac{1 - \eta + \chi \beta \phi}{RHS(n)} \tag{B.10}
\]

While \(LHS(1) = RHS(1) = 1\), \(\partial LHS/\partial n > 0\) while \(\partial RHS/\partial n < 0\), \(\forall n > 1\). Thus \(V_{i,ss}^s < V_{i,ss}^c\), \(\forall n > 1\).